BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Bull. Int. Math. Virtual Inst., Vol. **10**(1)(2020), 19-27
DOI: 10.7251/BIMVI2001019P

Formerly
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

NEW SETS IN IDEAL NANO TOPOLOGICAL SPACES

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ABSTRACT. In this paper focuss on $\star - nI_A$-sets, $\star - nI_\eta$-sets and $\star - nI_C$-sets in ideal nano topological spaces and certain properties of these investigated. We also investigated the notion of $R - nI$-open sets and discussed their relationships with other forms of nano ideal sets. Further, we have given an appropriate examples to understand the abstract concepts clearly.

1. Introduction and Preliminaries

An ideal I [**11**] on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.

(1) $A \in I$ and $B \subset A$ imply $B \in I$ and
(2) $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Let (X, τ) be a given topological space with an ideal I in X. If $\wp(X)$ is the family of all subsets of X, a set operator $(.)^\star : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$,

$$A^\star(I, \tau) = \{x \in X : U \cap A \notin I \text{ for every } U \in \tau(x)\}$$

where $\tau(x) = \{U \in \tau : x \in U\}$ [**2**]. The closure operator defined by $cl^\star(A) = A \cup A^\star(I, \tau)$ [**10**] is a Kuratowski closure operator which generates a topology $\tau^\star(I, \tau)$ called the $\star$-topology finer than τ. The topological space together with an ideal on X is called an ideal topological space or an ideal space denoted by (X, τ, I). We will simply write $A^\star$ for $A^\star(I, \tau)$ and $\tau^\star$ for $\tau^\star(I, \tau)$.

In this paper focuss on $\star$-nI_A-sets, $\star$-nI_η-sets and $\star$-nI_C-sets in ideal nano topological spaces and certain properties of these investigated. We also investigated the notion of R-nI-open sets and discussed their relationships with other forms of

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2010 *Mathematics Subject Classification.* 54C05, 54C08.
Key words and phrases. $\star$-nI_A-set, $\star$-nI_η-set, $\star$-nI_C-set, $nI_C^\star$-set, ideal nano topological space.

nano ideal sets. Further, we have given an appropriate examples to understand the abstract concepts clearly.

DEFINITION 1.1. ([**7**]) Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.

(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

DEFINITION 1.2. ([**3**]) Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(1) U and $\phi \in \tau_R(X)$,

(2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

(3) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

A nano topological space $(U, \mathcal{N})$ with an ideal I on U is called [**6**] an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$. $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$, denotes [**6**] the family of nano open sets containing x.

In future an ideal nano topological space $(U, \mathcal{N}, I)$ will be simply called a space.

DEFINITION 1.3. ([**5**]) A subset B of a space $(U, \mathcal{N}, I)$ is called $n\star$-closed if $B_n^\star \subseteq B$. The complement of a $n\star$-closed set is said to be $n\star$-open.

DEFINITION 1.4. (citerns18) A subset B of a space $(U, \mathcal{N}, I)$ is said to be

(1) α-nI-open if $B \subseteq n\text{-}int(n\text{-}cl^\star(n\text{-}int(B)))$.

(2) pre-nI-open if $B \subseteq n\text{-}int(n\text{-}cl^\star(B))$.

DEFINITION 1.5. ([**8**]) The pre-nI-closure of a subset B of an ideal nano topological space $(U, \mathcal{N}, I)$, denoted by $nI_pcl(B)$, is defined as the intersection of all pre-nI-closed sets of U containing B.

DEFINITION 1.6. ([**4**]) A subset A of a space $(U, \mathcal{N}, I)$, is called *semi**-nI-open if $B \subseteq n\text{-}cl(n\text{-}int^\star(B))$.

Definition 1.7. ([**9**]) A subset B of a space $(U, \mathcal{N}, I)$ is called a
(1) $t^{\#}$-nI-set if n-$int(B) = n$-$cl^{\star}(n$-$int(B))$.
(2) pre-nI-regular if B is pre-nI-open and $t^{\#}$-nI-set.

Definition 1.8. ([**1**]) A subset F of an nano topological space $(U, \mathcal{N})$ is said to be a nano locally closed set if $F = S \cap P$ where S is an n-open set and P is a n-closed set in U.

2. On $\star$-nI_A-sets and $\star$-nI_C-sets

Definition 2.1. A subset B of a space $(U, \mathcal{N}, I)$ is said to be a
(1) $\star$-nI_C-set if $B = S \cap P$, where S is $n\star$-open set and P is pre-nI-closed set in U.
(2) $\star$-nI_η-set if $B = S \cap P$, where S is $n\star$-open set and P is α-nI-closed set in U.
(3) $\star$-nI_A-set if $B = S \cap P$, where S is $n\star$-open set and $P = n$-$cl(n$-$int^{\star}(P))$.
(4) $nI_C^{\star}$-set if $B = S \cap P$, where S is $n\star$-open set and P is pre-nI-regular set in U.

Remark 2.1. Let $(U, \mathcal{N}, I)$ be an ideal nano topological space and $B \subseteq U$. The following diagram holds for B.

$$\begin{array}{ccc} nI_C^{\star}\text{-set} & \longrightarrow & \star\text{-}nI_C\text{-set} \\ & & \uparrow \\ \star\text{-}nI_A\text{-set} & \longrightarrow & \star\text{-}nI_\eta\text{-set} \end{array}$$

The following Examples show that these implications are not reversible.

Example 2.1. Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b\}, \{c\}\}$, $X = \{a, b\}$, $\mathcal{N} = \{\phi, U, \{a, b\}\}$ and $I = \{\phi\}$. Then $A = \{c\}$ is $\star$-nI_η-set but not an $\star$-nI_A-set.

Example 2.2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{b\}, \{d\}, \{a, c\}\}$, $X = \{c, d\}$, $\mathcal{N} = \{\phi, U, \{d\}, \{a, c\}, \{a, c, d\}$ and $I = \{\phi, \{d\}\}$. Then $A = \{b\}$ is $\star$-nI_C-set but not an $nI_C^{\star}$-set.

Example 2.3. In Example 2.2. Then $A = \{b, c\}$ is $\star$-nI_C-set but not an $\star$-nI_η-set.

Theorem 2.1. *For a subset B of a space $(U, \mathcal{N}, I)$, the following properties are equivalent.*
(1) *B is $\star$-nI_C-set and a semi*-nI-open set in U.*
(2) *$B = L \cap n$-$cl(n$-$int^{\star}(B))$ for an $n\star$-open set S.*

Proof. (1) $\Rightarrow$ (2): Assuming that B is $\star$-nI_C-set and semi*-nI-open set in U. Since B is $\star$-nI_C-set, then we have $B = S \cap P$, where S is $n\star$-open set and P is pre-nI-closed set in U. We have $B \subseteq P$, so n-$cl(n$-$int^{\star}(B)) \subseteq n$-$cl(n$-$int^{\star}(P))$. Since P is pre-nI-closed set in U, we have n-$cl(n$-$int^{\star}(P)) \subseteq P$. Since B is semi*-nI-open set in U, We have $P \subseteq n$-$cl(n$-$int^{\star}(B))$. It follows that $B = B \cap n$-$cl(n$-$int^{\star}(B)) = S \cap P \cap n$-$cl(n$-$int^{\star}(B)) = L \cap n$-$cl(n$-$int^{\star}(B))$.

(2) $\Rightarrow$ (1): Let $B = S \cap n$-$cl(n$-$int^{\star}(B))$ for $n\star$-open set S. We have $B \subseteq n$-$cl(n$-$int^{\star}(B))$. It follows that B is semi*-nI-open set in U. Since n-$cl(n$-$int^{\star}(B))$ is

a closed set, then $n\text{-}cl(n\text{-}int^{\star}(B))$ is pre-nI-closed set in U. Hence, B is $\star$-nI_C-set in U. □

THEOREM 2.2. *For a subset B of an ideal nano topological space $(U,\mathcal{N},I)$, the following properties are equivalent.*

(1) *B is $\star$-nI_A-set in U.*
(2) *B is $\star$-nI_η-set and a semi*-nI-open set in U.*
(3) *B is $\star$-nI_C-set and a semi*-nI-open set in U.*

PROOF. (1) ⇒ (2): Suppose that B is $\star$-nI_A-set in U. It follows that $B = S \cap P$, where S is $n\star$-open set and $P = n\text{-}cl(n\text{-}int^{\star}(P))$. This implies

$$\begin{aligned} B &= S \cap P \\ &= S \cap n\text{-}cl(n\text{-}int^{\star}(P)) \\ &= n\text{-}int^{\star}(S) \cap n\text{-}cl(n\text{-}int^{\star}(P)) \\ &\subseteq n\text{-}cl(n\text{-}int^{\star}(S)) \cap n\text{-}cl(n\text{-}int^{\star}(P)) \\ &\subseteq n\text{-}cl(n\text{-}int^{\star}(S) \cap n\text{-}int^{\star}(P)) \\ &= n\text{-}cl(n\text{-}int^{\star}(S \cap P)) \\ &= n\text{-}cl(n\text{-}int^{\star}(B)). \end{aligned}$$

Thus $B \subseteq n\text{-}cl(n - int^{\star}(B))$ and hence B is a semi*-nI-open set in U. Moreover, Remark 2.1, B is $\star$-nI_η-set in U.

(2) ⇒ (3): It follows from the fact that every $\star$-nI_η-set is $\star$-nI_C-set in U by Remark 2.1.

(3) ⇒ (1): Suppose that B is $\star$-nI_C-set and a semi*-nI-open set in U. By Theorem 2.1, $B = S \cap n\text{-}cl(n\text{-}int^{\star}(B))$ for a $n\star$-open set S. We have

$$n - cl(n - int^{\star}(n - cl(n - int^{\star}(B)))) = n - cl(n - int^{\star}(B)).$$

It follows that B is $\star$-nI_A-set in U. □

DEFINITION 2.2. A subset B of a space $(U,\mathcal{N},I)$ is said to be $\star$-nI_{gp}-open if $G \subseteq nI_p int(B)$ whenever $G \subseteq B$ and G is $n\star$-closed set in U where

$$nI_p int(B) = B \cap n - int(n - cl^{\star}(B)).$$

THEOREM 2.3. *For a subset B of a ideal nano topological space $(U,\mathcal{N},I)$, B is $\star$-nI_{gp}-closed if and only if $nI_p cl(B) \subseteq G$ whenever $B \subseteq G$ and G is $n\star$-open set in $(U,\mathcal{N},I)$.*

PROOF. Let B be $\star$-nI_{gp}-closed set in U. Suppose that $B \subseteq G$ and G is $n\star$-open set in $(U,\mathcal{N},I)$. Then $U - B$ is $\star$-nI_{gp}-open and $U - G \subseteq U - B$ where $U - G$ is $n\star$-closed. Since $U - B$ is $\star$-nI_{gp}-open, then we have

$$U - G \subseteq nI_p int(U - B),$$

where

$$nI_p int(U - B) = (U - B) \cap n - int(n - cl^{\star}(U - B)).$$

Since

$$\begin{aligned}(U-B)\cap n\text{-}int(n\text{-}cl^\star(U-B)) &= (U-B)\cap(U-n\text{-}cl(n\text{-}int^\star(B)))\\ &= U-(B\cup n\text{-}cl(n\text{-}int^\star(B))),\\ \text{then by } nI_pcl(B) &= B\cup n\text{-}cl(n\text{-}int^\star(B)),\\ (U-B)\cap n\text{-}int(n\text{-}cl^\star(U-B)) &= U-(B\cup n\text{-}cl(n\text{-}int^\star(B)))\\ &= U-nI_pcl(B).\end{aligned}$$

It follows that $nI_pint(U-B) = U-nI_pcl(B)$. Thus $nI_pcl(B) = U-nI_pint(U-B) \subseteq G$ and hence $nI_pcl(B) \subseteq G$.

The converse is similar. □

THEOREM 2.4. *Let $(U,\mathcal{N},I)$ be an ideal nano topological space and $F \subseteq U$. Then F is $\star$-nI_C-set in U if and only if $F = H \cap nI_pcl(F)$ for an $n\star$-open set H in U.*

PROOF. If F is $\star$-nI_C-set, then $F = H \cap P$ for an $n\star$-open set H and a pre-nI-closed set P. But then $F \subseteq P$ and so $F \subseteq nI_pcl(F) \subseteq P$. It follows that

$$\begin{aligned}F &= F\cap nI_pcl(F)\\ &= H\cap P\cap nI_pcl(F)\\ &= H\cap nI_pcl(F)\end{aligned}$$

Conversely, it is enough to prove that $nI_pcl(F)$ is a pre-nI-closed set. But $nI_pcl(F) \subseteq P$, for any pre-nI-closed set P containing F. So, $n\text{-}cl(n\text{-}int^\star(nI_pcl(F))) \subseteq n\text{-}cl(n\text{-}int^\star(P)) \subseteq P$. It follows that

$$n - cl(n - int^\star(nI_pcl(F))) \subseteq \cap F \subseteq P,$$

P is pre-nI-closed $P = nI_pcl(F)$. □

THEOREM 2.5. *Let $(U,\mathcal{N},I)$ be an ideal nano topological space and $B \subseteq U$. The following properties are equivalent.*

(1) *B is pre-nI-closed set in U.*
(2) *B is $\star$-nI_C-set and $\star$-nI_{gp}-closed set in U.*

PROOF. (1) ⇒ (2): It follows from the fact that any pre-nI-closed set in U is $\star$-nI_C-set and $\star$-nI_{gp}-closed set in U.

(2) ⇒ (1): Suppose that B is $\star$-nI_C-set and $\star$-nI_{gp}-closed set in U. Since B is $\star$-nI_C-set, then by Theorem 2.4, $B = H \cap nI_pcl(B)$ for $n\star$-open set H in $(U,\mathcal{N},I)$. Since $B \subseteq H$ and B is $\star$-nI_{gp}-closed set in U, then $nI_pcl(B) \subseteq H$. It follows that $nI_pcl(B) \subseteq H \cap nI_pcl(B) = B$. Thus, $B = nI_pcl(B)$ and hence B is pre-nI-closed. □

THEOREM 2.6. *Let $(U,\mathcal{N},I)$ be an nano ideal topological space and $B \subseteq U$. If B is $\star$-nI_C-set in U, then $nI_pcl(B)-B$ is pre-nI-closed set and $B\cup(U-nI_pcl(B))$ is pre-nI-open set in U.*

PROOF. Suppose that B is $\star$-nI_C-set in U. By Theorem 2.4, we have $B = L \cap nI_pcl(B)$ for $n\star$-open set S in U. It follows that

$$\begin{aligned} nI_pcl(B) - B &= nI_pcl(B) - (S \cap nI_pcl(B)) \\ &= nI_pcl(B) \cap (U - (S \cap nI_pcl(B))) \\ &= nI_pcl(B) \cap ((U - S) \cup (U - nI_pcl(B))) \\ &= (nI_pcl(B) \cap (U - S)) \cup (nI_pcl(B) \cap (U - nI_pcl(B))) \\ &= (nI_pcl(B) \cap (U - S)) \cup \phi \\ &= nI_pcl(B) \cap (U - S). \end{aligned}$$

Thus $nI_pcl(B) - B = nI_pcl(B) \cap (U - S)$ and hence $nI_pcl(B) - B$ is pre-nI-closed set. Moreover, since $nI_pcl(B) - B$ is pre-nI-closed set in U, then

$$U - (nI_pcl(B) - B) = (U - (nI_pcl(B) \cap (U - B)) = (U - nI_pcl(B)) \cup B$$

is pre-nI-open set. Thus,

$$U - (nI_pcl(B) - B) = (U - nI_pcl(B)) \cup B$$

is pre-nI-open set in U. □

3. On R-nI-open sets and $\star$-nI_A-sets

DEFINITION 3.1. A subset F of an ideal nano topological space $(U, \mathcal{N}, I)$ is said to be R-nI-open if $F = n\text{-}int(n\text{-}cl^\star(F))$. The complement of R-nI-open is R-nI-closed.

THEOREM 3.1. *For an ideal nano topological space $(U, \mathcal{N}, I)$ and a subset F of U, the following properties are equivalent:*

(1) *F is an R-nI-closed set,*
(2) *F is semi*-nI-open and n-closed.*

PROOF. (1) ⇒ (2) : Let F be an R-nI-closed set in U. Then we have $F = n\text{-}cl(n\text{-}int^\star(F))$. It follows that F is *semi**-nI-open and n-closed.

(2) ⇒ (1) : Assuming that F is a *semi**-nI-open set and a closed set in U. It follows that $F \subseteq n\text{-}cl(n\text{-}int^\star(F))$. Since F is n-closed, then we have

$$\begin{aligned} n\text{-}cl(n\text{-}int^\star(F)) &\subseteq n\text{-}cl(F) \\ &= F \subseteq n\text{-}cl(n\text{-}int^\star(F)). \end{aligned}$$

Thus, $F = n\text{-}cl(n\text{-}int^\star(F))$ and hence F is R-nI-closed. □

THEOREM 3.2. *For an ideal nano topological space $(U, \mathcal{N}, I)$ and a subset F of U, the following properties are equivalent:*

(1) *F is an R-nI-closed set,*
(2) *There exists a $n\star$-open set S such that $F = n\text{-}cl(S)$.*

PROOF. (2) $\Rightarrow$ (1) : Suppose that there exists a $n\star$-open set S such that $F = n\text{-}cl(S)$. Since $S = n\text{-}int^\star(S)$, then we have $n\text{-}cl(S) = n\text{-}cl(n\text{-}int^\star(S))$. It follows that

$$\begin{aligned} n\text{-}cl(n\text{-}int^\star(n\text{-}cl(S))) &= n\text{-}cl(n\text{-}int^\star(n\text{-}cl(n\text{-}int^\star(S)))) \\ &= n\text{-}cl(n\text{-}int^\star(S)) \\ &= n\text{-}cl(S). \end{aligned}$$

This implies $F = n\text{-}cl(S) = n\text{-}cl(n\text{-}int^\star(n\text{-}cl(S))) = n\text{-}cl(n\text{-}int^\star(F))$. Thus, $F = n\text{-}cl(n\text{-}int^\star(F))$ and hence F is an $R\text{-}nI$-closed set in U.

(1) $\Rightarrow$ (2) : Suppose that F is an $R\text{-}nI$-closed set in U. We have $F = n\text{-}cl(n\text{-}int^\star(F))$. We take $S = n\text{-}int^\star(F)$. It follows that S is a $n\star$-open set and $F = n\text{-}cl(S)$. □

THEOREM 3.3. *For an ideal nano topological space $(U, \mathcal{N}, I)$ and a subset F of U, F is semi*-nI-open if $F = S \cap P$ where S is an R-nI-closed set and $n\text{-}int(P)$ is a $n\star$-dense set.*

PROOF. Suppose that $F = S \cap P$ where S is an $R\text{-}nI$-closed set and $n\text{-}int(P)$ is a $n\star$-dense set. Then there exists a $n\star$-open set G such that $S = n\text{-}cl(G)$. We take $E = G \cap n\text{-}int(P)$. It follows that E is $n\star$-open and $E \subseteq F$. Moreover, we have $n\text{-}cl(E) = n\text{-}cl(G \cap n\text{-}int(P))$ and $n\text{-}cl(G \cap n\text{-}int(P)) \subseteq n\text{-}cl(G)$. Since $n\text{-}int(P)$ is $n\star$-dense, then we have

$$\begin{aligned} G &= G \cap n\text{-}cl^\star(n\text{-}int(P)) \\ &\subseteq n\text{-}cl^\star(G \cap n\text{-}int(P)) \\ &\subseteq n\text{-}cl(G \cap n\text{-}int(P)). \end{aligned}$$

It follows that $n\text{-}cl(G) \subseteq n\text{-}cl(G \cap n\text{-}int(P))$. Furthermore, we have

$$\begin{aligned} n\text{-}cl(E) &= n\text{-}cl(G \cap n\text{-}int(P)) \\ &\subseteq n\text{-}cl(G) \\ &= S \subseteq n\text{-}cl(G \cap n\text{-}int(P)) \\ &= n\text{-}cl(E). \end{aligned}$$

Thus, $E \subseteq F \subseteq S = n\text{-}cl(E)$. Hence, F is a *semi**-nI-open set in U. □

DEFINITION 3.2. The *semi**-nI-closure of a subset F of an ideal nano topological space $(U, \mathcal{N}, I)$, denoted by $s^\star_{nI}cl(F)$, is defined by the intersection of all *semi**-nI-closed sets of U containing F.

DEFINITION 3.3. Let $(U, \mathcal{N}, I)$ be an ideal topological space and $F \subseteq U$. F is called

(1) generalized *semi**-nI-closed ($gs^\star_{nI}$-closed) in $(U, \mathcal{N}, I)$ if $s^\star_{nI}cl(F) \subseteq E$ whenever $F \subseteq E$ and E is an n-open set in $(U, \mathcal{N}, I)$.

(2) generalized *semi**-nI-open ($gs^\star_{nI}$-open) in $(U, \mathcal{N}, I)$ if $U - F$ is a $gs^\star_{nI}$-closed set in $(U, \mathcal{N}, I)$.

THEOREM 3.4. *For a subset F of an ideal nano topological space $(U,\mathcal{N},I)$, F is $gs^{\star}_{nI}$-open if and only if $J \subseteq s^{\star}_{nI}$-$int(F)$ whenever $J \subseteq F$ and J is a n-closed set in $(U,\mathcal{N},I)$, where $s^{\star}_{nI}int(F) = F \cap n\text{-}cl(n\text{-}int^{\star}(F))$.*

PROOF. $(\Rightarrow)$: Suppose that F is a gs_{nI}-open set in U. Let $J \subseteq F$ and J be a n-closed set in $(U,\mathcal{N},I)$. It follows that $U-F$ is a $gs^{\star}_{nI}$-closed set and $U-F \subseteq U-J$ where $U-J$ is an n-open set. Since $U-F$ is $gs^{\star}_{nI}$-closed, then $s^{\star}_{nI}cl(U-F) \subseteq U-J$, where $s^{\star}_{nI}cl(U-F) = (U-F) \cup n\text{-}int(n\text{-}cl^{\star}(U-F))$. Since

$$\begin{aligned}(U-F) \cup n\text{-}int(n\text{-}cl^{\star}(U-F)) &= (U-F) \cup U - n\text{-}cl(n\text{-}int^{\star}(F)) \\ &= U - (F \cap n\text{-}cl(n\text{-}int^{\star}(F))), \\ \text{then } (U-F) \cup n\text{-}int(n\text{-}cl^{\star}(U-F)) &= U - (F \cap n\text{-}cl(n\text{-}int^{\star}(F))) \\ &= U - s^{\star}_{nI}int(F).\end{aligned}$$

It follows that $s^{\star}_{nI}cl(U-F) = U - s^{\star}_{nI}int(F)$. Thus,

$$J \subseteq U - s^{\star}_{nI}cl(U-F) = s^{\star}_{nI}int(F)$$

and hence $J \subseteq s^{\star}_{nI}int(F)$.

$(\Leftarrow)$: The converse is similar. □

THEOREM 3.5. *Let $(U,\mathcal{N},I)$ be an ideal nano topological space and $F \subseteq U$. The following properties are equivalent:*

(1) *F is an R-nI-open set,*
(2) *F is open and $gs^{\star}_{nI}$-closed.*

PROOF. (1) $\Rightarrow$ (2) : Let F be an R-nI-open set in U. Then we have $F = n\text{-}int(n\text{-}cl^{\star}(F))$. It follows that F is open and $semi^{\star}$-nI-closed in U. Thus, $s^{\star}_{nI}cl(F) \subseteq L$ whenever $F \subseteq L$ and L is an open set in $(U,\mathcal{N},I)$. Hence, F is a $gs^{\star}_{nI}$-closed set in U.

(2) $\Rightarrow$ (1) : Let F be n-open and $gs^{\star}_{nI}$-closed in U. We have $F \subseteq n\text{-}int(n\text{-}cl^{\star}(F))$. Since F is $gs^{\star}_{nI}$-closed and n-open, then we have $s^{\star}_{nI}cl(F) \subseteq F$. Since $s^{\star}_{nI}cl(F) = F \cup n\text{-}int(n\text{-}cl^{\star}(F))$, then $s^{\star}_{nI}cl(F) = F \cup n\text{-}int(n\text{-}cl^{\star}(F)) \subseteq F$. Thus, $n\text{-}int(n\text{-}cl^{\star}(F)) \subseteq F$ and $F \subseteq n\text{-}int(n\text{-}cl^{\star}(F))$. Hence, $F = n\text{-}int(n\text{-}cl^{\star}(F))$ and F is an R-nI-open set in U. □

REMARK 3.1. Any n-open set and any R-nI-closed set in U is an $\star$-nI_A-set in U. The reverse of this implication is not true in general as shown in the following example.

EXAMPLE 3.1. In Example 2.2. Then the set $A = \{b,d\}$ is an $\star$-nI_A-set but it is not n-open. The set $B = \{d\}$ is an $\star$-nI_A-set but it is not R-nI-closed.

REMARK 3.2. Any $\star$-nI_A-set is a nano locally closed set in U. The reverse implication is not true in general as shown in the following Example.

EXAMPLE 3.2. In Example 2.1, then the set $A = \{c\}$ is nano locally closed but it is not an $\star$-nI_A-set.

THEOREM 3.6. *Let $(U, \mathcal{N}, I)$ be an ideal nano topological space, $E \subseteq U$ and $F \subseteq U$. If E is a semi*-nI-open set and F is an n-open set, then $E \cap F$ is semi*-nI-open.*

PROOF. Suppose that E is a *semi**-nI-open set and F is an n-open set in U. It follows that

$$\begin{aligned} E \cap F &\subseteq n\text{-}cl(n\text{-}int^\star(E)) \cap F \\ &\subseteq n\text{-}cl(n\text{-}int^\star(E) \cap F) \\ &= n\text{-}cl(n\text{-}int^\star(E \cap F)). \end{aligned}$$

Thus, $E \cap F \subseteq n\text{-}cl(n\text{-}int^\star(E \cap F))$ and hence, $E \cap F$ is a *semi**-nI-open set in U. □

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Received by editors 15.05.2019; Revised version 20.05.2019; Available online 03.06.2019.

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