NEW SETS IN IDEAL NANO TOPOLOGICAL SPACES

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Abstract. In this paper focuss on $\ast - nI_A$-sets, $\ast - nI_\eta$-sets and $\ast - nI_C$-sets in ideal nano topological spaces and certain properties of these investigated. We also investigated the notion of $R - nI$-open sets and discussed their relationships with other forms of nano ideal sets. Further, we have given an appropriate examples to understand the abstract concepts clearly.

1. Introduction and Preliminaries

An ideal $I$ [11] on a topological space $(X, \tau)$ is a non-empty collection of subsets of $X$ which satisfies the following conditions.

1. $A \in I$ and $B \subset A$ imply $B \in I$ and
2. $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Let $(X, \tau)$ be a given topological space with an ideal $I$ in $X$. If $\varphi(X)$ is the family of all subsets of $X$, a set operator $(.)^* : \varphi(X) \to \varphi(X)$, called a local function of $A$ with respect to $\tau$ and $I$ is defined as follows: for $A \subset X$,

$$A^*(I, \tau) = \{x \in X : U \cap A \notin I \text{ for every } U \in \tau(x)\}$$

where $\tau(x) = \{U \in \tau : x \in U\}$ [2]. The closure operator defined by $cl^*(A) = A \cup A^*(I, \tau)$ [10] is a Kuratowski closure operator which generates a topology $\tau^*(I, \tau)$ called the $\ast$-topology finer than $\tau$. The topological space together with an ideal on $X$ is called an ideal topological space or an ideal space denoted by $(X, \tau, I)$. We will simply write $A^*$ for $A^*(I, \tau)$ and $\tau^*$ for $\tau^*(I, \tau)$.

In this paper focuss on $\ast - nI_A$-sets, $\ast - nI_\eta$-sets and $\ast - nI_C$-sets in ideal nano topological spaces and certain properties of these investigated. We also investigated the notion of $R - nI$-open sets and discussed their relationships with other forms of

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nano ideal sets. Further, we have given an appropriate examples to understand the abstract concepts clearly.

**Definition 1.1.** ([7]) Let $U$ be a non-empty finite set of objects called the universe and $R$ be an equivalence relation on $U$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(U, R)$ is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of $X$ with respect to $R$ is the set of all objects, which can be for certain classified as $X$ with respect to $R$ and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x$.

2. The upper approximation of $X$ with respect to $R$ is the set of all objects, which can be possibly classified as $X$ with respect to $R$ and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$.

3. The boundary region of $X$ with respect to $R$ is the set of all objects, which can be classified neither as $X$ nor as $\neg X$ with respect to $R$ and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

**Definition 1.2.** ([3]) Let $U$ be the universe, $R$ be an equivalence relation on $U$ and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. $U$ and $\emptyset \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on $U$ called the nano topology with respect to $X$ and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a $n$-open set is called $n$-closed.

A nano topological space $(U, \mathcal{N})$ with an ideal $I$ on $U$ is called [6] an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$. $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$, denotes [6] the family of nano open sets containing $x$.

In future an ideal nano topological space $(U, \mathcal{N}, I)$ will be simply called a space.

**Definition 1.3.** ([5]) A subset $B$ of a space $(U, \mathcal{N}, I)$ is called $n$-$\ast$-closed if $B_n^* \subseteq B$. The complement of a $n$-$\ast$-closed set is said to be $n$-$\ast$-open.

**Definition 1.4.** (cf.erns18) A subset $B$ of a space $(U, \mathcal{N}, I)$ is said to be

1. $\alpha$-$I$-open if $B \subseteq n$-$\mathrm{int}(n$-$\mathcal{cl}^\ast(n$-$\mathrm{int}(B)))$.
2. $\pre-nI$-open if $B \subseteq n$-$\mathrm{int}(n$-$\mathcal{cl}^\ast(B))$.

**Definition 1.5.** ([8]) The pre-$I$-closure of a subset $B$ of an ideal nano topological space $(U, \mathcal{N}, I)$, denoted by $nI_{c\ell}(B)$, is defined as the intersection of all pre-$I$-closed sets of $U$ containing $B$.

**Definition 1.6.** ([4]) A subset $A$ of a space $(U, \mathcal{N}, I)$, is called $\ semin{\ast}$-$I$-open if $B \subseteq n\mathcal{cl}(n$-$\mathrm{int}^\ast(B))$. 

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DEFINITION 1.7. ([9]) A subset $B$ of a space $(U, \mathcal{N}, I)$ is called
(1) $t^\# -nI$-set if $n\text{-}\text{int}(B) = n\text{-}\text{cl}^*(n\text{-}\text{int}(B))$.
(2) pre-$nI$-regular if $B$ is pre-$nI$-open and $t^\#-nI$-set.

DEFINITION 1.8. ([1]) A subset $F$ of an nano topological space $(U, \mathcal{N})$ is said to
be a nano locally closed set if $F = S \cap P$ where $S$ is an $n$-open set and $P$ is a
$n$-closed set in $U$.

2. On $\ast-nI_A$-sets and $\ast-nI_C$-sets

DEFINITION 2.1. A subset $B$ of a space $(U, \mathcal{N}, I)$ is said to be
(1) $\ast-nI_C$-set if $B = S \cap P$, where $S$ is $\ast$-open set and $P$ is pre-$nI$-closed set
in $U$.
(2) $\ast-nI_I$-set if $B = S \cap P$, where $S$ is $\ast$-open set and $P$ is $\alpha$-nI-closed set in
$U$.
(3) $\ast-nI_A$-set if $B = S \cap P$, where $S$ is $\ast$-open set and $P = n\text{-}\text{cl}(n\text{-}\text{int}^*(P))$.
(4) $nI_C^*$-set if $B = S \cap P$, where $S$ is $\ast$-open set and $P$ is pre-$nI$-regular set
in $U$.

REMARK 2.1. Let $(U, \mathcal{N}, I)$ be an ideal nano topological space and $B \subseteq U$.
The following diagram holds for $B$.

\[
\begin{array}{c}
nI_C^*-set \\ \downarrow \\
\ast-nI_C-set
\end{array}
\quad \text{and} \quad
\begin{array}{c}
\ast-nI_A-set \\ \downarrow \\
\ast-nI_I-set
\end{array}
\]

The following Examples show that these implications are not reversible.

EXAMPLE 2.1. Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b\}, \{c\}\}$, $X = \{a, b\}$, $\mathcal{N} = \{\phi, U, \{a, b\}\}$ and $I = \{\phi\}$. Then $A = \{c\}$ is $\ast-nI_I$-set but not an $\ast-nI_A$-set.

EXAMPLE 2.2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{b\}, \{d\}, \{a, c\}\}$, $X = \{c, d\}$, $\mathcal{N} = \{\phi, U, \{d\}, \{a, c\}, \{a, c, d\}\}$ and $I = \{\phi, \{d\}\}$. Then $A = \{b\}$ is $\ast-nI_C$-set but not an $nI_C^*$-set.

EXAMPLE 2.3. In Example 2.2. Then $A = \{b, c\}$ is $\ast-nI_C$-set but not an $\ast-nI_I$-set.

THEOREM 2.1. For a subset $B$ of a space $(U, \mathcal{N}, I)$, the following properties are
equivalent.

1. $B$ is $\ast-nI_C$-set and a semi$^\#$-nI-open set in $U$.
2. $B = L \cap n\text{-}\text{cl}(n\text{-}\text{int}^*(B))$ for an $n$-open set $S$.

PROOF. (1) $\Rightarrow$ (2): Assuming that $B$ is $\ast-nI_C$-set and semi$^\#$-nI-open set in
$U$. Since $B$ is $\ast-nI_C$-set, then we have $B = S \cap P$, where $S$ is $n$-open set and $P$ is
pre-$nI$-closed set in $U$. We have $B \subseteq P$, so $n\text{-}\text{cl}(n\text{-}\text{int}^*(B)) \subseteq n\text{-}\text{cl}(n\text{-}\text{int}^*(P))$.
Since $P$ is pre-$nI$-closed set in $U$, we have $n\text{-}\text{cl}(n\text{-}\text{int}^*(P)) \subseteq P$. Since $B$ is semi$^\#$-
$nI$-open set in $U$. We have $P \subseteq n\text{-}\text{cl}(n\text{-}\text{int}^*(B))$. It follows that $B = B \cap n\text{-}\text{cl}(n\text{-}\text{int}^*(B)) = S \cap P \cap n\text{-}\text{cl}(n\text{-}\text{int}^*(B)) = L \cap n\text{-}\text{cl}(n\text{-}\text{int}^*(B))$.

(2) $\Rightarrow$ (1): Let $B = S \cap n\text{-}\text{cl}(n\text{-}\text{int}^*(B))$ for $n$-open set $S$. We have $B \subseteq n\text{-}\text{cl}(n\text{-}\text{int}^*(B))$. It follows that $B$ is semi$^\#$-nI-open set in $U$. Since $n\text{-}\text{cl}(n\text{-}\text{int}^*(B))$ is
a closed set, then \( n-cl(n-int^*(B)) \) is pre-\( nI \)-closed in \( U \). Hence, \( B \) is \( *-nI_C \)-set in \( U \).

**Theorem 2.2.** For a subset \( B \) of an ideal nano topological space \((U, N, I)\), the following properties are equivalent.

1. \( B \) is \( *-nI_A \)-set in \( U \).
2. \( B \) is \( *-nI_p \)-set and a semi\(^*\)-\( nI \)-open set in \( U \).
3. \( B \) is \( *-nI_C \)-set and a semi\(^*\)-\( nI \)-open set in \( U \).

**Proof.** (1) \( \Rightarrow \) (2): Suppose that \( B \) is \( *-nI_A \)-set in \( U \). It follows that \( B = S \cap P \), where \( S \) is \( n^* \)-open set and \( P = n-cl(n-int^*(P)) \). This implies

\[
B = S \cap n-cl(n-int^*(P)) \\
= n-int^*(S) \cap n-cl(n-int^*(P)) \\
\subseteq n-cl(n-int^*(S)) \cap n-cl(n-int^*(P)) \\
\subseteq n-cl(n-int^*(S) \cap n-int^*(P)) \\
= n-cl(n-int^*(S \cap P)) \\
= n-cl(n-int^*(B)).
\]

Thus \( B \subseteq n-cl(n-int^*(B)) \) and hence \( B \) is a semi\(^*\)-\( nI \)-open set in \( U \). Moreover, Remark 2.1, \( B \) is \( *-nI_p \)-set in \( U \).

(2) \( \Rightarrow \) (3): It follows from the fact that every \( *-nI_q \)-set is \( *-nI_C \)-set in \( U \) by Remark 2.1.

(3) \( \Rightarrow \) (1): Suppose that \( B \) is \( *-nI_C \)-set and a semi\(^*\)-\( nI \)-open set in \( U \). By Theorem 2.1, \( B = S \cap n-cl(n-int^*(B)) \) for a \( n^* \)-open set \( S \). We have

\[
n - cl(n-int^*(n-cl(n-int^*(B)))) = n - cl(n-int^*(B)).
\]

It follows that \( B \) is \( *-nI_A \)-set in \( U \). \( \Box \)

**Definition 2.2.** A subset \( B \) of a space \((U, N, I)\) is said to be \( *-nI_p \)-open if \( G \subseteq nI_p\text{int}(B) \) whenever \( G \subseteq B \) and \( G \) is \( n^* \)-closed set in \( U \) where

\[
nI_p\text{int}(B) = B \cap n - \text{int}(n - cl^*(B)).
\]

**Theorem 2.3.** For a subset \( B \) of an ideal nano topological space \((U, N, I)\), \( B \) is \( *-nI_p \)-closed if and only if \( nI_p\text{cl}(B) \subseteq G \) whenever \( B \subseteq G \) and \( G \) is \( n^* \)-open set in \( (U, N, I) \).

**Proof.** Let \( B \) be \( *-nI_p \)-closed set in \( U \). Suppose that \( B \subseteq G \) and \( G \) is \( n^* \)-open set in \( (U, N, I) \). Then \( U - B \) is \( *-nI_p \)-open and \( U - G \subseteq U - B \) where \( U - G \) is \( n^* \)-closed. Since \( U - B \) is \( *-nI_p \)-open, then we have

\[
U - G \subseteq nI_p\text{int}(U - B),
\]

where

\[
nI_p\text{int}(U - B) = (U - B) \cap n - \text{int}(n - cl^*(U - B)).
\]
Since

\[(U - B) \cap n\text{-int}(n\text{-cl}^*(U - B)) = (U - B) \cap (U - n\text{-cl}(n\text{-int}^*(B))) = U - (B \cup n\text{-cl}(n\text{-int}^*(B))),\]

then by \(nI_p cl(B) = B \cup n\text{-cl}(n\text{-int}^*(B)),\)

\[(U - B) \cap n\text{-int}(n\text{-cl}^*(U - B)) = U - (B \cup n\text{-cl}(n\text{-int}^*(B))) = U - nI_p cl(B).\]

It follows that \(nI_p \text{int}(U - B) = U - nI_p cl(B).\) Thus \(nI_p cl(B) = U - nI_p \text{int}(U - B) \subseteq G\) and hence \(nI_p cl(B) \subseteq G.\)

The converse is similar. \(\square\)

**Theorem 2.4.** Let \((U, \mathcal{N}, I)\) be an ideal nano topological space and \(F \subseteq U.\) Then \(F\) is \(*nI_C\)-set in \(U\) if and only if \(F = H \cap nI_p cl(F)\) for an \(n\text{-open}\) set \(H\) in \(U.\)

**Proof.** If \(F\) is \(*nI_C\)-set, then \(F = H \cap P\) for an \(n\text{-open}\) set \(H\) and a pre-\(nI\)-closed set \(P.\) But then \(F \subseteq P\) and so \(F \subseteq nI_p cl(F) \subseteq P.\) It follows that

\[F = F \cap nI_p cl(F) = H \cap P \cap nI_p cl(F) = H \cap nI_p cl(F)\]

Conversely, it is enough to prove that \(nI_p cl(F)\) is a pre-\(nI\)-closed set. But \(nI_p cl(F) \subseteq P,\) for any pre-\(nI\)-closed set \(P\) containing \(F.\) So, \(n-cl(n\text{-int}^*(nI_p cl(F))) \subseteq n-cl(n\text{-int}^*(P)) \subseteq P.\) It follows that

\[n - cl(n - int^*(nI_p cl(F))) \subseteq \cap F \subseteq P,\]

\(P\) is pre-\(nI\)-closed \(P = nI_p cl(F).\) \(\square\)

**Theorem 2.5.** Let \((U, \mathcal{N}, I)\) be an ideal nano topological space and \(B \subseteq U.\) The following properties are equivalent.

1. \(B\) is pre-\(nI\)-closed set in \(U.\)
2. \(B\) is \(*nI_C\)-set and \(*nI_{gp}\)-closed set in \(U.\)

**Proof.** (1) \(\Rightarrow\) (2): It follows from the fact that any pre-\(nI\)-closed set in \(U\) is \(*nI_C\)-set and \(*nI_{gp}\)-closed set in \(U.\)

(2) \(\Rightarrow\) (1): Suppose that \(B\) is \(*nI_C\)-set and \(*nI_{gp}\)-closed set in \(U.\) Since \(B\) is \(*nI_C\)-set, then by Theorem 2.4, \(B = H \cap nI_p cl(B)\) for \(n\text{-open}\) set \(H\) in \((U, \mathcal{N}, I).\) Since \(B \subseteq H\) and \(B\) is \(*nI_{gp}\)-closed set in \(U,\) then \(nI_p cl(B) \subseteq H.\) It follows that \(nI_p cl(B) \subseteq H \cap nI_p cl(B) = B.\) Thus, \(B = nI_p cl(B)\) and hence \(B\) is pre-\(nI\)-closed. \(\square\)

**Theorem 2.6.** Let \((U, \mathcal{N}, I)\) be an ideal nano topological space and \(B \subseteq U.\) If \(B\) is \(*nI_C\)-set in \(U,\) then \(nI_p cl(B) - B\) is pre-\(nI\)-closed set and \(B \cup (U - nI_p cl(B))\) is pre-\(nI\)-open set in \(U.\)
Thus $nI \subseteq \text{set. Moreover, since } F = L \cap nI_{p}\text{cl}(B) \text{ for } \mathit{n}_{\star} \text{-open set } S \text{ in } U$. It follows that
\begin{align*}
nI_{p}\text{cl}(B) - B &= nI_{p}\text{cl}(B) - (S \cap nI_{p}\text{cl}(B)) \\
&= nI_{p}\text{cl}(B) \cap (U - (S \cap nI_{p}\text{cl}(B))) \\
&= nI_{p}\text{cl}(B) \cap ((U - S) \cup (U - nI_{p}\text{cl}(B))) \\
&= (nI_{p}\text{cl}(B) \cap (U - S)) \cup (nI_{p}\text{cl}(B) \cap (U - nI_{p}\text{cl}(B))) \\
&= (nI_{p}\text{cl}(B) \cap (U - S)) \cup \phi \\
&= nI_{p}\text{cl}(B) \cap (U - S).
\end{align*}
Thus $nI_{p}\text{cl}(B) - B = nI_{p}\text{cl}(B) \cap (U - S)$ and hence $nI_{p}\text{cl}(B) - B$ is pre-$nI$-closed set. Moreover, since $nI_{p}\text{cl}(B) - B$ is pre-$nI$-closed set in $U$, then
\begin{align*}
U - (nI_{p}\text{cl}(B) - B) &= (U - (nI_{p}\text{cl}(B) \cap (U - B))) = (U - nI_{p}\text{cl}(B)) \cup B
\end{align*}
is pre-$nI$-open set. Thus,
\begin{align*}
U - (nI_{p}\text{cl}(B) - B) &= (U - nI_{p}\text{cl}(B)) \cup B
\end{align*}
is pre-$nI$-open set in $U$. \hfill \Box

3. On $R\mathit{nI}$-open sets and $\mathit{nI}_{A}$-sets

**Definition 3.1.** A subset $F$ of an ideal nano topological space $(U, \mathcal{N}, I)$ is said to be $R\mathit{nI}$-open if $F = n\text{-int}(n\text{-cl}^{*}(F))$. The complement of $R\mathit{nI}$-open is $R\mathit{nI}$-closed.

**Theorem 3.1.** For an ideal nano topological space $(U, \mathcal{N}, I)$ and a subset $F$ of $U$, the following properties are equivalent:

1. $F$ is an $R\mathit{nI}$-closed set.
2. $F$ is semi$^*$-$nI$-open and $n$-closed.

**Proof.** (1) $\Rightarrow$ (2) : Let $F$ be an $R\mathit{nI}$-closed set in $U$. Then we have $F = n\text{-cl}(n\text{-int}^{*}(F))$. It follows that $F$ is semi$^*$-$nI$-open and $n$-closed.

(2) $\Rightarrow$ (1) : Assuming that $F$ is a semi$^*$-$nI$-open set and a closed set in $U$. It follows that $F \subseteq n\text{-cl}(n\text{-int}^{*}(F))$. Since $F$ is $n$-closed, then we have
\begin{align*}
n\text{-cl}(n\text{-int}^{*}(F)) &\subseteq n\text{-cl}(F) \\
&= F \subseteq n\text{-cl}(n\text{-int}^{*}(F)).
\end{align*}
Thus, $F = n\text{-cl}(n\text{-int}^{*}(F))$ and hence $F$ is $R\mathit{nI}$-closed. \hfill \Box

**Theorem 3.2.** For an ideal nano topological space $(U, \mathcal{N}, I)$ and a subset $F$ of $U$, the following properties are equivalent:

1. $F$ is an $R\mathit{nI}$-closed set.
2. There exists a $n$-$\star$-open set $S$ such that $F = n\text{-cl}(S)$.
Proof. (2) $\Rightarrow$ (1): Suppose that there exists a $n^\ast$-open set $S$ such that $F = n-cl(S)$. Since $S = n-int^\ast(S)$, then we have $n-cl(S) = n-cl(n-int^\ast(S))$. It follows that
\[
\begin{align*}
n-cl(n-int^\ast(n-cl(S))) &= n-cl(n-int^\ast(n-cl(n-int^\ast(S)))) \\
&= n-cl(n-int^\ast(S)) \\
&= n-cl(S).
\end{align*}
\]
This implies $F = n-cl(S) = n-cl(n-int^\ast(n-cl(S))) = n-cl(n-int^\ast(F))$. Thus, $F = n-cl(n-int^\ast(F))$ and hence $F$ is an $R-nI$-closed set in $U$.

(1) $\Rightarrow$ (2): Suppose that $F$ is an $R-nI$-closed set in $U$. We have $F = n-cl(n-int^\ast(F))$. We take $S = n-int^\ast(F)$. It follows that $S$ is a $n^\ast$-open set and $F = n-cl(S)$.

Theorem 3.3. For an ideal nano topological space $(U, N, I)$ and a subset $F$ of $U$, $F$ is semi-$nI$-open if $F = S \cap P$ where $S$ is an $R-nI$-closed set and $n-int(P)$ is a $n^\ast$-dense set.

Proof. Suppose that $F = S \cap P$ where $S$ is an $R-nI$-closed set and $n-int(P)$ is a $n^\ast$-dense set. Then there exists a $n^\ast$-open set $G$ such that $S = n-cl(G)$. We take $E = G \cap n-int(P)$. It follows that $E$ is $n^\ast$-open and $E \subseteq F$. Moreover, we have $n-cl(E) = n-cl(G \cap n-int(P))$ and $n-cl(G \cap n-int(P)) \subseteq n-cl(G)$. Since $n-int(P)$ is $n^\ast$-dense, then we have
\[
\begin{align*}
G &= G \cap n-cl^\ast(n-int(P)) \\
&\subseteq n-cl^\ast(G \cap n-int(P)) \\
&\subseteq n-cl(G \cap n-int(P)).
\end{align*}
\]
It follows that $n-cl(G) \subseteq n-cl(G \cap n-int(P))$. Furthermore, we have
\[
\begin{align*}
n-cl(E) &= n-cl(G \cap n-int(P)) \\
&\subseteq n-cl(G) \\
&= S \subseteq n-cl(G \cap n-int(P)) \\
&= n-cl(E).
\end{align*}
\]
Thus, $E \subseteq F \subseteq S = n-cl(E)$. Hence, $F$ is a semi-$nI$-open set in $U$.

Definition 3.2. The semi-$nI$-closure of a subset $F$ of an ideal nano topological space $(U, N, I)$, denoted by $s_{nI}^\ast cl(F)$, is defined by the intersection of all semi-$nI$-closed sets of $U$ containing $F$.

Definition 3.3. Let $(U, N, I)$ be an ideal topological space and $F \subseteq U$. $F$ is called

1. generalized semi-$nI$-closed $(gs_{nI}^\ast$-closed) in $(U, N, I)$ if $s_{nI}^\ast cl(F) \subseteq E$ whenever $F \subseteq E$ and $E$ is an $n$-open set in $(U, N, I)$.

2. generalized semi-$nI$-open $(gs_{nI}^\ast$-open) in $(U, N, I)$ if $U - F$ is a $gs_{nI}^\ast$-closed set in $(U, N, I)$.
THEOREM 3.4. For a subset $F$ of an ideal nano topological space $(U, \mathcal{N}, I)$, $F$ is $gs_{nI}$-open if and only if $J \subseteq s_{nI}^*\text{int}(F)$ whenever $J \subseteq F$ and $J$ is a $n$-closed set in $(U, \mathcal{N}, I)$, where $s_{nI}^*\text{int}(F) = F \cap n\text{-cl}(n\text{-int}(F))$.

**Proof.** ($\Rightarrow$) Suppose that $F$ is a $gs_{nI}$-open set in $U$. Let $J \subseteq F$ and $J$ be a $n$-closed set in $(U, \mathcal{N}, I)$. It follows that $U - F$ is a $gs_{nI}$-closed set and $U - F \subseteq U - J$ where $U - J$ is an $n$-open set. Since $U - F$ is $gs_{nI}$-closed, then $s_{nI}^*\text{cl}(U - F) \subseteq U - J$, where $s_{nI}^*\text{cl}(U - F) = (U - F) \cup \text{n-int}(n\text{-cl}(U - F))$. Since

$$(U - F) \cup \text{n-int}(n\text{-cl}(U - F)) = (U - F) \cup U - n\text{-cl}(n\text{-int}(F))$$

$$= U - (F \cap n\text{-cl}(n\text{-int}(F))),$$

then $(U - F) \cup \text{n-int}(n\text{-cl}(U - F)) = U - (F \cap n\text{-cl}(n\text{-int}(F)))$

$$= U - s_{nI}^*\text{int}(F).$$

It follows that $s_{nI}^*\text{cl}(U - F) = U - s_{nI}^*\text{int}(F)$. Thus,

$$J \subseteq U - s_{nI}^*\text{cl}(U - F) = s_{nI}^*\text{int}(F)$$

and hence $J \subseteq s_{nI}^*\text{int}(F)$.

($\Leftarrow$) : The converse is similar. \hfill $\square$

THEOREM 3.5. Let $(U, \mathcal{N}, I)$ be an ideal nano topological space and $F \subseteq U$. The following properties are equivalent:

1. $F$ is an $R-nI$-open set,
2. $F$ is open and $gs_{nI}$-closed.

**Proof.** (1) $\Rightarrow$ (2) : Let $F$ be an $R-nI$-open set in $U$. Then we have $F = \text{n-int}(n\text{-cl}(F))$. It follows that $F$ is open and $\text{semi}^*\text{-nI}$-closed in $U$. Thus, $s_{nI}^*\text{cl}(F) \subseteq L$ whenever $F \subseteq L$ and $L$ is an open set in $(U, \mathcal{N}, I)$. Hence, $F$ is a $gs_{nI}$-closed set in $U$.

(2) $\Rightarrow$ (1) : Let $F$ be $n$-open and $gs_{nI}$-closed in $U$. We have $F \subseteq \text{n-int}(n\text{-cl}(F))$. Since $F$ is $gs_{nI}$-closed and $n$-open, then we have $s_{nI}^*\text{cl}(F) \subseteq F$. Since $s_{nI}^*\text{cl}(F) = F \cup n\text{-int}(n\text{-cl}(F))$, then $s_{nI}^*\text{cl}(F) = F \cup n\text{-int}(n\text{-cl}(F)) \subseteq F$. Thus, $n\text{-int}(n\text{-cl}(F)) \subseteq F$ and $F \subseteq n\text{-int}(n\text{-cl}(F))$. Hence, $F = n\text{-int}(n\text{-cl}(F))$ and $F$ is an $R-nI$-open set in $U$. \hfill $\square$

**Remark 3.1.** Any $n$-open set and any $R-nI$-closed set in $U$ is an $\ast-nI_A$-set in $U$. The reverse of this implication is not true in general as shown in the following example.

**Example 3.1.** In Example 2.2. Then the set $A = \{b, d\}$ is an $\ast-nI_A$-set but it is not $n$-open. The set $B = \{d\}$ is an $\ast-nI_A$-set but it is not $R-nI$-closed.

**Remark 3.2.** Any $\ast-nI_A$-set is a nano locally closed set in $U$. The reverse implication is not true in general as shown in the following Example.

**Example 3.2.** In Example 2.1, then the set $A = \{c\}$ is nano locally closed but it is not an $\ast-nI_A$-set.
Theorem 3.6. Let \((U,N,I)\) be an ideal nano topological space, \(E \subseteq U\) and \(F \subseteq U\). If \(E\) is a semi\(^*\)-\(nI\)-open set and \(F\) is an \(n\)-open set, then \(E \cap F\) is semi\(^*\)-\(nI\)-open.

Proof. Suppose that \(E\) is a semi\(^*\)-\(nI\)-open set and \(F\) is an \(n\)-open set in \(U\). It follows that
\[
E \cap F \subseteq \text{n-cl}(n\text{-int}^*(E)) \cap F \\
\subseteq \text{n-cl}(n\text{-int}^*(E) \cap F) \\
= \text{n-cl}(n\text{-int}^*(E \cap F)).
\]
Thus, \(E \cap F \subseteq \text{n-cl}(n\text{-int}^*(E \cap F))\) and hence, \(E \cap F\) is a semi\(^*\)-\(nI\)-open set in \(U\).

References


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