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TRI-IDEALS OF SEMIRINGS

M. Murali Krishna Rao

Abstract. In this paper, we introduce the notion of a tri-ideal as a generalization of bi-quasi-interior ideal,quasi-interior ideal,bi-interior ideal, bi-quasi ideal, quasi ideal, interior ideal,left(right) ideal and ideal of a semiring. Then, we study the properties of tri-ideals of a semiring and characterize the tri-simple semiring using tri-ideals of a semiring.

1. Introduction

The algebraic structure play a prominent role in mathematics with wide range of applications. Generalization of ideals of algebraic structures and ordered algebraic structure plays a very remarkable role and also necessary for further advance stuctiondies and application of various algebraic structures. During 1950-1980, the concepts of bi-ideals, quasi ideals and interior ideals were studied by many mathematicians and during 1950-2019, the applications of these ideals only studied by mathematicians.

Between 1980 and 2016 there have been no new generalization of these ideals of algebraic structures. Then the author [**22, 23, 24, 21, 29, 25, 30, 27, 26**] introduced and studied bi quasi ideals, bi-interior ideals, bi quasi interior ideals, quasi interior ideals and weak interior ideals of $\Gamma-$semirings, semirings, $\Gamma-$semigroups,semigroups as a generalization of bi-ideal, quasi ideal and interior ideal of algebraic structures and characterized regular algebraic structures as weil as simple algebraic structures using these ideals. The notion of a semiring was introduced by Vandiver [**34**] in 1934, but semirings had appeared in earlier studies on the theory of ideals of rings. Semiring is a generalization of ring but also of

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a generalization of distributive lattice.semirings are structurally similar to semigroups than to rings. Semiring theory has many applications in other branches of mathematics.

As a generalization of ring, the notion of a Γ-ring was introduced by Nobusawa [**15**] in 1964. In 1995, M. Murali Krishna Rao [**16, 17, 18, 20**] introduced the notion of Γ-semiring as a generalization of Γ-ring, ternary semiring and semiring. Sen [**31**] introduced the notion of a Γ-semigroup as a generalization of semigroup. The notion of a ternary algebraic system was introduced by Lehmer [**13**] in 1932. Dutta and Sardar [**2**] introduced the notion of operator semirings of Γ-semiring. Lister [**14**] introduced ternary ring. Murali Krishna Rao and Venkateswarlu [**19, 30, 27, 28**] studied regular Γ-incline, field Γ-semiring and derivations.

Many mathematicians introduced various generalizations of concept of ideals in algebraic structures, proved important results and characterizations of regular algebraic structures using bi-ideals, quasi ideals and simple algebraic structures using interior ideals. Henriksen [**4**] and Shabir and Batod [**32**] studied ideals in semirings. We know that the notion of a one sided ideal of any algebraic structure is a generalization of notion of an ideal. The quasi ideals are generalization of left ideal and right ideal whereas the bi-ideals are generalization of quasi ideals.

In 1952, the concept of bi-ideals was introduced by Good and Hughes [**3**] for semigroups. The notion of bi-ideals in rings and semirings were introduced by Lajos and Szasz [**11, 12**]. Bi-ideal is a special case of (m-n) ideal. Steinfeld [**33**] first introduced the notion of quasi ideals for semigroups and then for rings. Iseki [**7, 5, 6, 8**] introduced the concept of quasi ideal for a semiring. In 1995, M. Murali Krishna Rao [**16, 17, 18, 20**] introduced the notion of $\Gamma-$semiring as a generalization of $\Gamma-$ ring, ternary semiring and semiring. Murali Krishna Rao and Venkateswarlu [**19, 30, 27, 26**] studied regular $\Gamma-$incline, field $\Gamma-$semiring and derivations. Quasi ideals, bi-ideals in $\Gamma-$semirings studied by Jagtap and Pawar [**9, 10**]. Murali Krishna Rao [**20, 22, 23, 24, 21, 29, 25**] introduced the notion of left (right) bi-quasi ideal, the notion of bi-interior ideal and the notion of bi quasi-interior ideal of $\Gamma-$semiring as a generalization of ideal of $\Gamma-$semiring, studied their properties and characterized the simple $\Gamma-$semiring and regular $\Gamma-$semiring using these ideals.

In this paper, we introduce the notion of tri-ideals as a generalization of quasi ideal, bi-ideal, interior ideal, left(right) ideal and ideal of semiring and study the properties of tri-ideals of a semiring.

2. Preliminaries

In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1. ([**1**]) A set M together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called semiring provided

(i) addition is a commutative operation.

(ii) multiplication distributes over addition both from the left and from the right.
(iii) there exists $0 \in M$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in M$.

Example 2.1. Let M be the set of all natural numbers. Then $(M,\ max,\ min)$ is a semiring.

Definition 2.2. Let M be a semiring. If there exists $1 \in M$ such that $a \cdot 1 = 1 \cdot a = a$, for all $a \in M$, is called an unity element of M then M is said to be semiring with unity.

Definition 2.3. An element a of a semiring M is called a regular element if there exists an element b of M such that $a = aba$.

Definition 2.4. A semiring M is called a regular semiring if every element of S is a regular element.

Definition 2.5. An element a of a semiring M is called a multiplicatively idempotent (an additively idempotent) element if $aa = a(a + a = a)$.

Definition 2.6. An element b of a semiring M is called an inverse element of a of M if $ab = ba = 1$.

Definition 2.7. A semiring M is called a division semiring if for each non-zero element of M has multiplication inverse.

Definition 2.8. A non-empty subset A of a semiring M is called

(i) a subsemiring of M if $(A, +)$ is a subsemigroup of $(M, +)$ and $AA \subseteq A$.
(ii) a quasi ideal of M if A is a subsemiring of M and $AM \cap MA \subseteq A$.
(iii) a bi-ideal of M if A is a subsemiring of M and $AMA \subseteq A$.
(iv) an interior ideal of M if A is a subsemiring of M and $MAM \subseteq A$.
(v) a left (right) ideal of M if A is a subsemiring of M and $MA \subseteq A(AM \subseteq A)$.
(vi) an ideal if A is a subsemiring of $M, AM \subseteq A$ and $MA \subseteq A$.
(vii) a $k-$ideal if A is a subsemiring of $M, AM \subseteq A, MA \subseteq A$ and $x \in M,\ x + y \in A, y \in A$ then $x \in A$.
(viii) a bi-interior ideal of M if A is a subsemiring of M and $MBM \cap BMB \subseteq B$.
(ix) a left bi-quasi ideal (right bi-quasi ideal) of M if A is a subsemiring of M and $MA \cap AMA \subseteq A$ $(AM \cap AMA \subseteq A)$.
(x) a left quasi-interior ideal (right quasi-interior ideal) of M if A is a subsemigroup of $(M, +)$ and $MAMA \subseteq A$ $(AMAM \subseteq A)$.
(xi) a bi-quasi-interior ideal of M if A is a subsemiring of M and $BMBMB \subseteq B$.

Definition 2.9. A semiring M is called a left bi-quasi simple semiring if M has no left bi-quasi ideal other than M itself.

3. Tri-ideals of semirings

In this section, we introduce the notion of tri-ideal as a generalization of bi-ideal, quasi-ideal and interior ideal of a semiring and study the properties of tri-ideal of a semiring.Throughout this paper$M$ is a semiring with unity element.

DEFINITION 3.1. A non-empty subset B of a semiring M is said to be right tri-ideal of M if B is a subsemiring of M and $BBMB \subseteq B$.

DEFINITION 3.2. A non-empty subset B of a semiring M is said to be left tri-ideal of M if B is a subsemiring of M and $BMBB \subseteq B$.

DEFINITION 3.3. A non-empty subset B of a semiring M is said to be tri-ideal of M if B is a subsemiring of M and $B$is a left and a right tri-ideal of M.

Remark: A tri-ideal of a semiring M need not be quasi-ideal, interior ideal, bi-interior ideal. and bi-quasi ideal of a semiring M.

EXAMPLE 3.1. (i) If $M = \left\{ \begin{pmatrix} a & 0 \\ c & 0 \end{pmatrix} \mid a, b, c \in Q \right\}$, then M is a semiring with respect to usual addition of matrices and ternary operation is defined as usual matrix multiplication and $A = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \mid 0 \neq a, 0 \neq b \in Q \right\}$. Then A is not a left tri-ideal of semiring M.

(ii) If $M = \left\{ \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} \mid a, b, c \in Q \right\}$ then M is a semiring with respect to usual addition of matrices and usual matrix multiplication and $A = \{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid 0 \neq a, 0 \neq b \in Q\}$. Then A is not a bi-ideal and A is a left tri-ideal of the semiring M

(iii) If $M = \left\{ \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} \mid a, b, c \in Q \right\}$ then M is a semiring with respect to usual addition of matrices and ternary operation is defined as usual matrix multiplication and $A = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid 0 \neq a, 0 \neq b \in Q \right\}$. Then A is not a bi-ideal and A is a left tri-ideal of the semiring M.

In the following theorem, we mention some important properties and we omit the proofs since they are straight forward.

THEOREM 3.1. *Let M be a semring. Then the following are hold.*

(1) *Every left ideal is a tri-ideal of M.*
(2) *Every right ideal is a tri-ideal of M.*
(3) *Every quasi ideal is a tri-ideal of M.*
(4) *Every ideal is a tri-ideal of M.*
(5) *If L is a left ideal and R is a right ideal of M then $B = R \cap L$ is a tri-ideal of M.*
(6) *If L is a left ideal and R is a right ideal of a semiring M then $B = RL$ is a tri-ideal of M.*
(7) *Let M be a semiring and B be a subsemiring of M.If $MMMB \subseteq B$ and$BMMM \subseteq B$ then B is a tri-ideal of M.*
(8) *Let M be a semiring and B be a subsemiring of M. If $MMM \subseteq B$ then B is a left tri-ideal of M.*

THEOREM 3.2. *If B be a left bi-quasi ideal of a semiring M, then B is a tri-ideal of M.*

PROOF. Suppose B is a left bi-quasi ideal of the semiring M. Then $BMB \subseteq MB$. We have $BMBB \subseteq BMB$ Therefore $BMBB \subseteq MB \cap BMB \subseteq B$ Hence B is a left tri-ideal of M. Similarly we can show that B is a right tri-ideal of M. Hence B is a tri-ideal of M. □

COROLLARY 3.1. *If B be a right bi-quasi ideal of a semiring M, then B is a tri- ideal of M.*

COROLLARY 3.2. *If B be a bi-quasi ideal of a semiring M, then B is a tri-ideal of M.*

THEOREM 3.3. *If B be a bi–interior ideal of a semiring M, then B is a left tri-ideal of M.*

PROOF. Suppose B is a bi–interior ideal of the semiring M. Then

$$MBM \cap BMB \subseteq B, \text{ and } BMBB \subseteq MBM \cap BMB \subseteq B.$$

Hence B is a left tri ideal of M. □

COROLLARY 3.3. *If B be a bi–interior ideal of a semiring M, then B is a right tri-ideal of M.*

COROLLARY 3.4. *If B be a bi–interior ideal of a semiring M, then B is a tri-ideal of M.*

THEOREM 3.4. *If B is a subsemiring of a semiring M and $MBB \subseteq B$, then B is a left tri- ideal of M.*

THEOREM 3.5. *Every bi-ideal of a semiring M is a left tri-ideal of a semiring M.*

PROOF. Let B be a bi-ideal of the semiring M. Then $BMBB \subseteq BMB \subseteq B$. Therefore $BMBB \subseteq B$. Hence every bi-ideal of a semiring M is a left tri-ideal of the semiring M. □

COROLLARY 3.5. *Every bi-ideal of a semiring M is a right tri-ideal of M.*

COROLLARY 3.6. *Every bi-ideal of a semiring M is a tri-ideal of M.*

THEOREM 3.6. *Every bi-quasi interior ideal of a semiring M is a left tri-ideal of a semiring M.*

PROOF. Let B be a bi-quasi interior ideal of the semiring M. Then $BMBMB \subseteq B$. Therefore $BMBB \subseteq BMBMB \subseteq B$. This completes the proof. □

COROLLARY 3.7. *Every bi-quasi interior ideal of a semiring M is a right tri-ideal of a semiring M.*

COROLLARY 3.8. *Every bi-quasi interior ideal of a semiring M is a tri-ideal of a semiring M.*

THEOREM 3.7. *Every interior ideal of a semiring M is a left tri-ideal of M.*

Proof. Let I be an interior ideal of the semiring M. Then $IMII \subseteq MIM \subseteq I$. Hence I is a left tri-ideal of the semiring M. □

Corollary 3.9. *Every interior ideal of a semiring M is a right tri-ideal of M.*

Corollary 3.10. *Every interior ideal of a semiring M is a tri-ideal of M.*

Theorem 3.8. *Let M be a semiring and B be a subsemiring of M and $B = BB$. Then B is a left tri-ideal of M if and only if there exist left ideal L and a right ideal R such that $RL \subseteq B \subseteq R \cap L$.*

Proof. Suppose B is a tri-ideal of the semiring$M$. Then $BMBB \subseteq B$. Let $R = BM$ and $L = MB$. Then R and L are a right ideal and a left ideal of M respectively. Therefore $RL \subseteq B \subseteq R \cap L$.

Conversely suppose that there exist R and L are a right ideal and a left ideal of M respectively such that $R\Gamma L \subseteq B \subseteq R \cap L$. Then

$$BMBB \subseteq (R \cap L)M(R \cap L)(R \cap L) \subseteq RL \subseteq B.$$

Hence B is a left tri-ideal of M. □

Corollary 3.11. *Let M be a semiring and B be a subsemiring of M and$B = BB$.Then B is a right tri-ideal of M if and only if there exist left ideal L and a right ideal R of M such that $RL \subseteq B \subseteq R \cap L$.*

Corollary 3.12. *Let M be a semiring and B be a subsemiring of M and$B = BB$.Then B is a tri-ideal of M if and only if there exist left ideal L and a right ideal R of M such that $RL \subseteq B \subseteq R \cap L$.*

Theorem 3.9. *The intersection of a left tri-ideal B of a semiring M and a right ideal A of M is always a left tri-ideal of M.*

Proof. Suppose $C = B \cap A$. Then $CMCC \subseteq BMBB \subseteq B$ and $CMCC \subseteq AMAA \subseteq A$. Since A is a left ideal of M, we have $CMCC \subseteq B \cap A = C$. Hence the intersection of a left tri-ideal B of the semiring M and a left ideal A of M is always a left tri-ideal of M. □

Corollary 3.13. *The intersection of a right tri-ideal B of a semiring M and a right ideal A of M is always a right tri-ideal of M.*

Corollary 3.14. *The intersection of a tri-ideal B of a semiring M and an ideal A of M is always a tri-ideal of M.*

Theorem 3.10. *Let A and C be left tri- ideals of a semiring M, $B = AC$ and B is an additively subsemigroup of M. If $AA = A$ then B is a left tri-ideal of M.*

Proof. Let A and C be left tri-ideals of the semiring M and $B = AC$. Then

$$BB = ACAC = ACAAC \subseteq AMAAC \subseteq AC = B.$$

Therefore $B = AC$ is a subsemiring of M and

$$BMBB = ACMACAC \subseteq AMAC \subseteq AC = B.$$

Hence B is a left tri-ideal of M. □

THEOREM 3.11. *Let A and C be subsemirings of a semiring M and $B = AC$ and B is additively subsemigroup of M. If A is the left ideal of M, then B is a tri-ideal of M.*

PROOF. Let A and C be subsemirings of M and $B = AC$. Suppose A is the left ideal of M. Then $BB = ACAC \subseteq AC = B$. Thus $BMBB = ACMACAC \subseteq AC = B$. Hence B is a left tri-ideal of M. □

COROLLARY 3.15. *Let A and C be subsemirings of a semiring M and $B = AC$and B is additively subsemigroup of M. If C is a right ideal then B is a right tri-ideal of M.*

THEOREM 3.12. *Let M be a semiring and T be a non-empty subset of M. If subsemiring B of M containing $TMTT$ and $B \subseteq T$, then B is a left tri-ideal of semiring M.*

PROOF. Let B be a subsemiring of M containing $TMTT$. Then $BMB \subseteq TMTT \subseteq B$. Therefore $BMBB \subseteq B$. Hence B is a left tri-ideal of M. □

THEOREM 3.13. *Let B be a tri-ideal of a semiring M and I be an interior ideal of M. Then $B \cap I$ is a left tri-ideal of M.*

PROOF. Suppose B is the tri-ideal of M and I is an interior ideal of M. Obviously $B \cap I$ is subsemiring of M. Then

$$(B \cap I)M(B \cap I)(B \cap I) \subseteq BMBB \subseteq B$$
$$(B \cap I)M(B \cap I)(B \cap I) \subseteq IMI \subseteq I$$

Therefore $(B \cap I)M(B \cap I)(B \cap I) \subseteq B \cap I$. Hence $B \cap I$ is a left tri-ideal of M. □

COROLLARY 3.16. *Let B be a tri-ideal of a semiring M and I be an interior ideal of M. Then $B \cap I$ is a right tri-ideal of M.*

COROLLARY 3.17. *Let B be a tri-ideal of a semiring M and I be an interior ideal of M. Then $B \cap I$ is a tri-ideal of M.*

THEOREM 3.14. *Let M be a semiring and T be a subsemiring of M. Then every subsemiring of T containing $TMTT$ is a left tri-ideal of M.*

PROOF. Let C be a subsemiring of T containing $TMTT$. Then

$$CMCC \subseteq TMTT \subseteq C.$$

Hence C is a left tri-ideal of M . □

THEOREM 3.15. *The intersection of left tri-ideals $\{B_\lambda \mid \lambda \in A\}$ of a semiring M is a left tri-ideal of M.*

PROOF. Let $B = \bigcap_{\lambda \in A} B_\lambda$. Then B is a subsemiring of M. Since B_λ is a left tri-ideal of M, we have $B_\lambda M B_\lambda B_\lambda \subseteq B_\lambda$, for all $\lambda \in A$. Then $\cap B_\lambda M \cap B_\lambda \cap B_\lambda \cap B_\lambda \subseteq \cap B_\lambda$ and thus $BMBB \subseteq B$. Hence B is a left tri-ideal of M. □

THEOREM 3.16. *Let B be a left tri-ideal of asemiring M, $e \in B$, $eB \subseteq B$ and e be $\beta-$idempotent. Then eB is a left tri-ideal of M.*

PROOF. Let B be a left tri-ideal of the semiring M. Suppose $x \in B \cap e\Gamma M$. Then $x \in B$ and $x = ey, y \in M$. Thus $x = ey = eey = e(ey) = ex \in eB$. Therefore $B \cap eM \subseteq eB$, $eB \subseteq B$ and $eB \subseteq eM$. Thus $eB \subseteq B \cap eM$ and $eB = B \cap eM$. Hence eB is a left tri-ideal of M. □

COROLLARY 3.18. *Let M be a semiring M and e be idempotent. Then eM and Me are left tri-ideal and right tri-ideal of M respectively.*

THEOREM 3.17. *Let M be a semiring. If $M = Ma$, for all $a \in M$. Then every left tri-ideal of M is a quasi ideal of M.*

PROOF. Let B be a left tri-ideal of the semiring M and $a \in B$. Then $Ma \subseteq MB$ and $M \subseteq MB \subseteq M$. Thus $MB = M$ and $BM = BMB \subseteq BMBB \subseteq B$. So, $MB \cap BM \subseteq M \cap BM \subseteq BM \subseteq B$. Therefore B is a quasi ideal of M. Hence the theorem. □

4. Tri-simple semiring, regular semiring and minimal tri- ideals of a semiring

In this section,we introduce the notion of left tri-simple semiring and characterize the left tri-simple semiring using left tri- ideals of semiring and study the properties of minimal left tri- ideals of a semiring

DEFINITION 4.1. A semiring M is a left (right) simple semiring if M has no proper left (right) ideals of M.

DEFINITION 4.2. A semiring M is said to be simple semiring if M has no proper ideals of M.

DEFINITION 4.3. A semiring M is said to be bi- simple semiring if M has no proper bi- ideals of M.

DEFINITION 4.4. A semiring M is said to be left(right) tri- simple semiring if M has no left(right) tri-ideal other than M itself.

DEFINITION 4.5. A semiring M is said to be tri- simple semiring if M has no tri-ideal other than M itself.

THEOREM 4.1. *If M is a division semiring then M is a tri- simple semiring.*

PROOF. Let B be a proper left tri-ideal of the division semiring M, $x \in M$ and $0 \neq a \in B$. Since M is a division semiring, there exists $b \in M$ such that $ab = 1$. Then $abx = x = xab$. Therefore $x \in BM$ and $M \subseteq BM$. We have $BM \subseteq M$. Hence $M = BM$. Similarly we can prove $MB = M$.

$$M = MB = BMB = BMBB \subseteq B, M \subseteq B.$$

Therefore $M = B$ and $M = BM = BBMB \subseteq B, M \subseteq B$. Therefore $M = B$. Hence division semiring M has no proper -tri-ideals. □

Theorem 4.2. *Let M be a left simple semiring. Every left tri-ideal of M is a right ideal of M.*

Proof. Let M be a left simple semiring and B be a left tri-ideal of M. Then $BMBB \subseteq B$ and MB is a left ideal of M. Since M is a left simple semiring, we have $MB = M$. Therefore $BMBB \subseteq B$. Thus $BM \subseteq B$. Hence the theorem. □

Corollary 4.1. *Let M be a right simple semiring. Every right tri-ideal is a left ideal of M.*

Corollary 4.2. *Let M be a left and a right simple semiring. Every tri-ideal is an ideal of M.*

Theorem 4.3. *Let M be a semiring. M is a left tri-simple semiring if and only if $< a >= M$, for all $a \in M$ where $< a >$ is the smallest left tri-ideal generated by a.*

Proof. Let M be a semiring. Suppose M is the left tri-simple semiring, $a \in M$ and $B = Ma$. Then B is a left ideal of M. Therefore, by Theorem 4.1, B is a left tri-ideal of M. Therefore $B = M$. Hence $Ma = M$, for all $a \in M$. Then $Ma \subseteq < a > \subseteq M$ and $M \subseteq < a > \subseteq M$. Therefore $M =< a >$.

Suppose $< a >$ is the smallest left tri-ideal of M generated by a, $< a >= M$, A is the left tri-ideal and $a \in A$. Then $< a > \subseteq A \subseteq M$ and $M \subseteq A \subseteq M$. Therefore $A = M$. Hence M is a left tri-ideal simple semiring. □

Theorem 4.4. *If semiring M is a left simple semiring then every left tri-ideal of M is a right ideal of M.*

Proof. Let B be a left tri-ideal of the left simple semiring$M$. Then MB is a left ideal of M and $MB \subseteq M$. Therefore $MB = M$. Then $BMBB \subseteq B$ and $BMB \subseteq B$. Thus $BM \subseteq B$. Hence every left tri-ideal is a right ideal of M. □

Corollary 4.3. *If semiringl M is right simple semiring then every right tri-ideal of M is a left ideal of M.*

Corollary 4.4. *Every tri-ideal of left and right simple semiring M is an ideal of M.*

Theorem 4.5. *Let M be a semiring and B be a left tri-ideal of M. Then B is a minimal left tri-ideal of M if and only if B is a left tri- simple subsemiring of M.*

Proof. Let B be a minimal left tri-ideal of the semiring M and C be a left tri-ideal of B. Then $CBCC \subseteq C$ and $CBCC$ is a left tri-ideal of M. Since C is a tri-ideal of B, we hve $CBCC = B$ and $B = CBCC \subseteq C$. Thus $B = C$.

Conversely suppose that B is a left tri-simple subsemiring of M. Let C be a left tri-ideal of M and $C \subseteq B$. Then $CBCC \subseteq CMCC \subseteq BMBB \subseteq B$. Therefore C is a left tri-ideal of B. Thus $B = C$ since B is a left tri-simple subsemiring of M. Hence B is a minimal left tri-ideal of M. □

Corollary 4.5. *Let M be a semiring and B be a right tri-ideal of M. Then B is a minimal right tri-ideal of M if and only if B is a right tri- simple subsemiring of M.*

COROLLARY 4.6. *Let M be a semiring and B be a tri-ideal of M. Then B is a minimal tri-ideal of M if and only if B is a tri- simple subsemiring of M.*

THEOREM 4.6. *Let M be a commutative idempotent semiring. Then M is a regular semiring if and only if $BMBB = B$ for all tri- ideals B of M.*

PROOF. Suppose M is a regular commutative idempotent semiring, B is a tri-ideal of M and $x \in B$. Then $BMBB \subseteq B$ and there exists $y \in M, x = xyxx \in BMBB$. Therefore $x \in BMBB$. Hence $BMBB = B$.

Conversely suppose that $BMBB = B$ for all tri- ideals B of M. Let $B = R \cap L$, where R and L are ideals of M. Then B is tri- ideal of M. Therefore

$$(R \cap L)M(R \cap L)(R \cap L) = R \cap L$$

and $R \cap L = (R \cap L)M(R \cap L)(R \cap L)$. Thus $RMRL \subseteq RL \subseteq R \cap L$ since $RL \subseteq L$ and $RL \subseteq R$. Therefore $R \cap L = RL$. Hence M is a regular semiring. □

5. Conclusion

As a further generalization of ideals, we introduced the notion of a tri-ideal of semiring as a generalization of ideal, left ideal, right ideal, bi-ideal,quasi ideal, bi-quasi ideal, bi-interior ideal,bi-quasi interior ideal and interior ideal of semiring and studied some of their properties. We introduced the notion of tri- simple semiring and characterized the tri-simple semiring of semiring. We proved every bi-quasi ideal of semiring and bi-interior ideal of semiring are tri-ideals and studied some of the properties of bi-interior ideals of semiring. In continuity of this paper, we study prime tri-ideals,prime, maximal and minimal tri-ideals of semiring.

References

[1] P. J. Allen. A fundamental theorem of homomorphism for semirings. *Proc. Amer. Math. Soc.*, **21**(2)(1969), 412–416.
[2] T. K. Dutta and S. K. Sardar. On the operator semirings of a $\Gamma-$semiring. *Southeast Asian Bull. Math.*, **26**(2)(2002), 203–213.
[3] R. A. Good and D. R. Hughes. Associated groups for a semigroup. *Bull. Amer. Math. Soc.*, **58** (1952), 624–625.
[4] M. Henriksen. Ideals in semirings with commutative addition. *Notices Amer. Math. Soc.*, **6**(1958), 321.
[5] K. Iseki. Ideal theory of semiring. *Proc. Japan Acad.*, **32**(8((1956), 554–559.
[6] K. Iseki. Ideals in semirings. *Proc. Japan Acad.*, **34**(1)(1958), 29–31.
[7] K. Iseki. Quasi-ideals in semirings without zero. *Proc. Japan Acad.*, **34**(2)(1958), 79–84.
[8] K. Izuka. On the Jacobson radical of a semiring. *Tohoku Math. J.*, **11**(3)(1959), 409–421.
[9] R. D. Jagatap and Y. S. Pawar. Quasi-ideals and minimal quasi-ideals in $\Gamma-$semirings. *Novi Sad J. Math.*, **39**(2)(2009), 79–87.
[10] R. D. Jagatap and Y. S. Pawar. Bi-ideals in $\Gamma-$ semirings. *Bull. Int. Math. Virtual Inst.*, **6**(2)(2016), 169–179.
[11] S. Lajos. On the bi-ideals in semigroups. *Proc. Japan Acad.*, **45**(8)(1969), 710–712.
[12] S. Lajos and F. A. Szasz. On the bi-ideals in associative ring. *Proc. Japan Acad.*, **46**(6)(1970), 505–507.
[13] H. Lehmer. A ternary analogue of Abelian groups. *Amer. J. Math.*, **54**(2)(1932), 329–338.
[14] W. G. Lister. Ternary rings. *Trans. Amer. Math. Soc.*, **154** (1971), 37–55.
[15] N. Nobusawa. On a generalization of the ring theory. *Osaka. J. Math.*, **1**(1)(1964), 81–89.

[16] M. M. K. Rao. $\Gamma-$semirings - I. *Southeast Asian Bull. Math.*, **19**(1)(1995), 49–54.
[17] M. M. K. Rao. Γ-semirings - II. *Southeast Asian Bull. Math.*, **21**(3)(1997), 281–287.
[18] M. M. K. Rao. The Jacobson radical of Γ-semiring. *Southeast Asian Bull. Math.*, **23**(1)(1999), 127–134
[19] M. M. K. Rao and B. Venkateswarlu. Regular $\Gamma-$incline and field $\Gamma-$semiring. *Novi Sad J. Math.*, **45**(2)(2015), 155–171.
[20] M. M. K. Rao. $\Gamma-$ semiring with identity. *Disc. Math. Gen. Alg. Appl.*, **37**(2)(2017), 189–207.
[21] M. M. K. Rao. Bi-quasi-ideals and fuzzy bi-quasi ideals of $\Gamma-$semigroups. *Bull. Int. Math. Virtual Inst.*, **7**(2)(2017), 231–242.
[22] M. M. K. Rao. Ideals in ordered $\Gamma-$semirings. *Disc. Math. Gen. Alg. and Appl.*, **38**(1)(2018), 47–68.
[23] M. M. K. Rao. Bi-interior Ideals in semigroups. *Disc. Math. Gen. Alg. Appl.*, **38**(1)(2018), 69–78.
[24] M. M. K. Rao. Left bi-quasi ideals of semirings. *Bull. Int. Math. Virtual Inst.* , **8**(1)(2018), 45–53.
[25] M. M. K. Rao. A study of bi-quasi interior ideal as a new generalization of ideal of generalization of semiring. *J. Int. Math. Virtual Inst.*, **9**(1)(2019), 19–35.
[26] M. M. K. Rao. Quasi - interior ideals of semirings. *Bull. Int. Math. Virtual Inst.*, **9**(1)(2019), 231–242.
[27] M. M. K. Rao and B. Venkateswarlu. On generalized right derivations of incline. *J. Int. Math. Virtual Inst.*, **6**(2016), 31–47.
[28] M. M. K. Rao and B. Venkateswarlu. Right derivation of ordered $\Gamma-$semirings. *Disc. Math. Gen. Alg. Appl.*,**36** (2016), 209–221.
[29] M. M. K. Rao, B. Venkateswarlu and N. Rafi. Left bi-quasi-ideals of $\Gamma-$semirings. *Asia Paci. J. Math.*, **4**(2)(2017), 144–153.
[30] M. M. K. Rao and B. Venkateswarlu. Bi-interior ideals in $\Gamma-$ semirings. *Disc. Math. Gen. Alg. and Appl.*, **38**(2)(2018), 209–221.
[31] M. K. Sen. On $\Gamma-$semigroup. *Proceedings of International Conference of Algebra and its Application*, 1981 (pp. 301–308). Lecture Notes in Pure and Appl. Math. 91, Dekker, New York, 1984.
[32] A. M. Shabir, A. Ali and S. Batod. A note on quasi ideal in semirings. *Southeast Asian Bull. Math.*, **27**(5)(2004), 923–928.
[33] O. Steinfeld. Uher die quasi ideals, Von halb gruppen. *Publ. Math., Debrecen*, **4** (1956), 262–275.
[34] H. S. Vandiver. Note on a simple type of algebra in which cancellation law of addition does not hold. *Bull. Amer. Math. Soc.(N.S.)*, **40**(12)(1934), 914–920.

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Department of Mathematics, GIT, GITAM University, Visakhapatnam - 530 045, A.P., India
E-mail address: mmarapureddy@gmail.com