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SOFT SEMIGROUPS

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Abstract. In this paper, we study $\Gamma-$semigroup M as soft semigroups over M.We prove that if e is a strongly idempotent left zeroid then soft semigroup (h, Γ) is a soft regular semigroup over M.

1. Introduction

As a generalization of ring, the notion of a $\Gamma-$ring was introduced by N. Nobusawa [**21**] in 1964. In 1995, M. Murali Krishna Rao [**14, 15, 16**] introduced the notion of $\Gamma-$semiring as a generalization of $\Gamma-$ring, ternary semiring and semiring. The notion of ternary algebraic system was introduced by Lehmer [**10**] in 1932. Lister [**11**] introduced ternary ring. Dutta & Kar [**6**] introduced the notion of ternary semiring which is a generalization of ternary ring and semiring. In 1981, Sen [**20, 21**] introduced the notion of $\Gamma-$semigroup as a generalization of semigroup. Clifford and Miller [**17**] studied zeroid elements in semigroups. D. F. Dawson [**5**] studied semigroups having left or right zeroid elements. The zeroid of a semigroup was introduced by Bourne and Zassenhaus [**3**]. M. Murali Krishna Rao and K. Rajendra Kumar [**17**] studied left zeroid and tight zeroid elements of $\Gamma-$semiring . M. Murali Krishna Rao [**18**] studied $\Gamma-$ semiring M as soft semiring over M. In 1999, Molodtsov [**12**] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Aktas and Cagman [**2**] defined the notion of soft groups. In 2008, F. Feng, Y. B. Jun and X. Zhao [**7**] defined soft semiring and several related notions to establish a connection between soft sets and semirings. Soft rings are defined by U. Acar, F. Koyuncu and B. Tanay [**1**] and J. Ghosh, B. Dinda and T. K. Samanta [**8**] initiated the study of fuzzy soft rings and

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fuzzy soft ideals. In this paper, we study $\Gamma-$semigroup M as soft semigroups over M.

2. Preliminaries

In this section, we recall some definitions introduced by the pioneers in this field earlier.

Definition 2.1. Let $(M, +)$ and $(\Gamma, +)$ be non-empty sets. Then we call M a $\Gamma-$semigroup, if there exists a mapping $M \times \Gamma \times M \to M$ (images of (x, α, y) will be denoted by $x\alpha y, x, y \in M, \alpha \in \Gamma$) such that it satisfies $x\alpha(y\beta z) = (x\alpha y)\beta z$. for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$

Every semigroup M is a $\Gamma-$semigroup with $\Gamma = M$ and ternary operation $x\gamma y$ defined as the usual semigroup multiplication.

Definition 2.2. A $\Gamma-$semigroup M is called a $\Gamma-$group if M_α is a group, for some $\alpha \in \Gamma$.

Definition 2.3. A $\Gamma-$semigroup M is said to be commutative $\Gamma-$semigroup if $x\alpha y = y\alpha x$, for all $x, y \in M$ and $\alpha \in \Gamma$.

Definition 2.4. Let M be a $\Gamma-$semigroup. An element of M is said to be idempotent of M if it is α idempotent for some $\alpha \in \Gamma$.

Definition 2.5. Let M be a $\Gamma-$semigroup. If every element of M is an idempotent of M,then M is said to be an idempotent $\Gamma-$semigroup.

Definition 2.6. Let M be a $\Gamma-$semigroup. An element $a \in M$ is said to be regular element of M if there exist $x \in M, \alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$.

Definition 2.7. Let M be a $\Gamma-$semigroup. If every element of M is regular, then M is said to be regular $\Gamma-$semigroup.

Definition 2.8. An element x of a semigroup M is called a left zeroid (right zeroid) if for each $y \in M$, there exists $a \in M$ such that $ay = x$ $(ya = x)$.

Definition 2.9. A non-empty subset A of a $\Gamma-$semigroup M is called a $\Gamma-$subsemigroup M if $a\alpha b \in A$ for all $a, b \in A$ and $\alpha \in \Gamma$.

Definition 2.10. ([**11**]) A subsemigroup I of a $\Gamma-$semigroup M is said to be left (right) ideal of M if $M\Gamma I \subseteq I$ $(I\Gamma M \subseteq I)$. If I is both left and right ideal then I is called an ideal of $\Gamma-$semigroup M.

Definition 2.11. Let U be an initial universe set and E be the set of parameters. Let $P(U)$ denotes the power set of U. A pair (f, E) is called a soft set over U where f is a mapping given by $f : E \to P(U)$.

Definition 2.12. Let U be an initial universe set and E be the set of parameters. Let $A \subseteq E$. A pair (f, A) is called a fuzzy soft set over U where f is a mapping given by $f : A \to I^U$ where I^U denotes the collection of all fuzzy subsets of U.

Definition 2.13. Let X be a group and (f, A) be a soft set over X. Then (f, A) is said to be soft group over X if and only if $f(a)$ is a subgroup of X for each $a \in A$.

Definition 2.14. For a soft set (f, A), the set $\{x \in A \mid f(x) \neq \emptyset\}$ is called a Support of (f, A) denoted by $Supp(f, A)$. If $\text{Supp}(f, A) \neq \emptyset$ then (f, A) is called non null soft set.

Definition 2.15. Let $(f, A), (g, B)$ be two soft sets over U then (f, A) is said to be soft subset of (g, B) denoted by $(f, A) \subseteq (g, B)$ if $A \subseteq B$ and $f(a) \subseteq g(a)$ for all $a \in A$.

Definition 2.16. Let $(f, A), (g, B)$ be non null soft sets. The intersection of soft sets (f, A) and (g, B) is denoted by $(f, A) \cap (g, B) = (h, C)$ where $C = A \cup B$ is defined as

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B; \\ g_c, & \text{if } c \in B \setminus A; \\ f_c \cap g_c, & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.17. ([**7**]) Let $(f, A), (g, B)$ be non null soft sets. The Union of soft sets (f, A) and (g, B) is denoted by $(f, A) \cup (g, B) = (h, C)$ where $C = A \cup B$ is defined as

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B; \\ g_c, & \text{if } c \in B \setminus A; \\ f_c \cup g_c, & \text{if } c \in A \cap B. \end{cases}$$

3. Soft semigroups

In this section, we study $\Gamma-$semigroup M as soft semigroup over M..

Definition 3.1. Let M be a semigroup and E be a parameter set, $A \subseteq E$. Let f be a mapping given by $f : A \to P(M)$ where $P(M)$ is the power set of M. Then (f, A) is called a soft semigroup over M if and only if for each $a \in A$, $f(a)$ is subsemigroup of M. i.e.

$$x, y \in f(a) \Rightarrow xy \in f(a).$$

Definition 3.2. Let M be a semigroup and E be a parameter set, $A \subseteq E$. Let f be a mapping given by $f : A \to P(M)$. Then (f, A) is called a soft left(right) ideal over M if and only if for each $a \in A, f(a)$ is a left(right) ideal of S. i.e.,

$$x \in f(a)r \in M \Rightarrow rx(xr) \in f(a).$$

Definition 3.3. Let M be a semigroup and E be a parameter set, $A \subseteq E$ and $f : A \to P(M)$. Then (f, A) is called a soft ideal over M if and only if for each $a \in A, f(a)$ is an ideal of M. i.e.,

$$x \in f(a), r \in M \Rightarrow rx \in f(a) \text{ and } xr \in f(a).$$

Theorem 3.1. *Every $\Gamma-$semigroup M is a soft semigroup over M.*

Proof. Let M be a $\Gamma-$semigroup and $\alpha \in \Gamma$. Define a mapping $* : M \times M \rightarrow M$ such that $a * b = a\alpha b$, for all $a, b \in M$. Then $(M, *)$ is a semigroup of M. It is denoted by M_α. Define $F : \Gamma \rightarrow \mathbb{P}(M)$, $\mathbb{P}(M)$ is a power set of M by $F(\alpha) = M_\alpha$. Hence $\Gamma-$semigroup M can be considered as a soft semigroup over M. It is denoted by (F, Γ). □

Converse of this theorem need not be true.

Example 3.1. Let $M = \{0, 1\}$ and $\Gamma = \{\alpha, \beta\}$. We define operations with the following tables:

α	0	1
0	0	0
1	1	1

β	0	1
0	0	1
1	1	1

.

Then (F, Γ) is a soft semigroup but M is not a $\Gamma-$semigroup.

Example 3.2. Let $M = \{0, 1\}$ and $\Gamma = \{\alpha, \beta\}$. We define operations with the following tables:

α	0	1
0	0	0
1	1	1

β	0	1
0	0	0
1	1	1

.

Then M is a $\Gamma-$semigroup and $\Gamma-$semigroup M is a soft semigroup over M. It is denoted by (F, Γ).

Theorem 3.2. *Let M be a $\Gamma-$semigroup with zero. Then the following are equivalent.*

(i) *M is a $0-$simple $\Gamma-$semigroup*
(ii) *M_α is a $0-$simple semigroup, for any $\alpha \in \Gamma$*
(iii) *M_α is a $0-$simple semigroup, for some $\alpha \in \Gamma$*
(iv) *$M\alpha a\beta M = M$, for any $a \in M \setminus \{0\}$, $\alpha, \beta, \in \Gamma$*

Proof. Let M be a $\Gamma-$semigroup with zero.

$(i) \Rightarrow (ii)$: Since $M\alpha a\alpha M$ is an ideal of M_α, for every $\alpha \in \Gamma, a \in M \setminus \{0\}$. Then $M\alpha a\alpha M = M$. Therefore M_α is a $0-$simple semigroup.

$(ii) \Rightarrow (iii)$: Obvious.

$(iii) \Rightarrow (iv)$: Suppose M_α is $0-$simple, for some $\alpha \in \Gamma$. Then

$$(M\gamma a\beta M)\alpha M \subseteq M\gamma a\beta M \text{ and } M\alpha(M\gamma a\beta M) \subseteq M\gamma a\beta M, \gamma, \beta \in \Gamma.$$

Thus $M\gamma a\beta M$ is an ideal of M_α. Therefore $M\gamma a\beta M = M$, for any $a \in M \setminus \{0\}$.

$(iv) \Rightarrow (i)$: Let A be an ideal of M containing an element a. Then $M\alpha a\beta M \subseteq A$. Thus $M \subseteq A$ and finally $M = A$. Hence M is a $0-$simple $\Gamma-$semigroup. □

Corollary 3.1. *Let M be a $\Gamma-$semigroup. Then M_α is a $0-$simple semigroup for some $\alpha \in \Gamma$ if and only if (F, Γ) is a soft $0-$simple semigroup.*

Definition 3.4. Let S be a $\Gamma-$semigroup. Let f be a mapping given by $f : A \rightarrow P(S)$ where $P(S)$ is the power set of S. Then (f, Γ) is called a soft $\Gamma-$group over S if and only if for each $a \in \Gamma$, $f(a)$ is a group.

Theorem 3.3. *Let M be a $\Gamma-$semigroup. Then the following are equivalent*

(i) *M is a $\Gamma-$group*
(ii) *M_α is a group, for some $\alpha \in \Gamma$*
(iii) *M_α is a group, for all $\alpha \in \Gamma$*

Proof. Let M be a $\Gamma-$semigroup.
$(i) \Rightarrow (ii)$: From the definition of $\Gamma-$group.
$(ii) \Rightarrow (iii)$: Let M_α be a group, for some $\alpha \in \Gamma$ and $\beta \in \Gamma, a \in M$. Then $(a\beta M)\alpha M \subseteq a\beta M$ or $M\alpha(M\beta a) \subseteq M\beta a$. Therefore $a\beta M$ and $M\beta a$ are right ideal and left ideal respectively of M_α. Since M_α is a group, we have $a\beta M = M\beta a = M$. Therefore M_β is a group.
$(iii) \Rightarrow (i)$: Obvious. □

Corollary 3.2. *Let M be a $\Gamma-$semigroup. Then M_α is a group for some $\alpha \in \Gamma$ if and only if (f, Γ) is a soft group over M .*

Definition 3.5. An element x of a Γ-semigroup M is called a left zeroid (right zeroid) if for each $y \in M$, $\alpha \in \Gamma$, there exists $a \in M$ such that $a\alpha y = x$ $(y\alpha a = x)$.

Definition 3.6. Let M be a $\Gamma-$semigroup. An element $a \in M$ is said to be strongly idempotent if $a = a\alpha a$; for all $\alpha \in \Gamma$.

Theorem 3.4. *Let M be a Γ-semigroup. If e is a left zeroid of a semigroup M_α, for some $\alpha \in \Gamma$ then e is a left zeroid of a soft semigroup (f, Γ) over M .*

Proof. Suppose e is a left zeroid of the semigroup M_α, $\alpha, \beta \in \Gamma$, $x \in M$. Therefore $x\beta x \in M$. Then there exists $z \in M$ such that $z\alpha(x\beta x) = e$, since e is a left zeroid of the semigroup M_α. From this follows $(z\alpha x)\beta x = e$. Hence e is a left zeroid of the semigroup M_β. Therefore e is a left zeroid of the semigroup M_β for all $\beta \in \Gamma$. Hence the Theorem. □

Theorem 3.5. *Let M be a Γ-semigroup , x is a left zeroid element of a semigroup M_α, for some $\alpha \in \Gamma$ and e is a strongly idempotenti. Then e is a right identity of a soft semigroup (f, Γ) over$M$. i.e.$x\alpha e = x$ for all $\alpha \in \Gamma$.*

Proof. Let x be a left zeroid element of the semigroup M_α, for some $\alpha \in \Gamma$. Then there exists $a \in M$ such that $a\alpha e = x$. Therefore $x\alpha e = a\alpha e\alpha e = a\alpha e = x$. Therefore $x\alpha e = x$ for all $\alpha \in \Gamma$. Hence$e$ is a right identity of the soft semigroup (F, Γ) over M. □

Let M be a $\Gamma-$semigroup and $\alpha \in \Gamma$. and $a \in M$ Define a mapping $* : M \times M \to M$ such that $a * b = a\alpha b$, for all $b \in M$. Then $(M, +, *)$ is a subsemigroup of M. It is denoted by $a\alpha M$. Define $g : \Gamma \to \mathbb{P}(M)$, $\mathbb{P}(M)$ is a power set of M by $g(\alpha) = a\alpha M$. Then (g, Γ) is a soft semigroup over M.

Theorem 3.6. *Let M be a $\Gamma-$semigroup. If e is a strongly idempotent, $\alpha \in \Gamma, x \in M$, then e is the left identity of a soft semigroup (g, Γ) over M.*

Proof. Let $e\alpha x \in e\alpha M$. Then $e\alpha e\alpha x = e\alpha x$. Hence e is the left identity of a soft semigroup (g, Γ) over M. □

Theorem 3.7. *Let M be a $\Gamma-$semigroup. If e is a strongly idempotent left zeroid of M , $\alpha \in \Gamma$, then (g, Γ) is a soft group over M.*

Proof. By Theorem 3.6, e is the left identity of the soft semigroup (g, Γ) over M. Suppose $e\alpha b \in e\alpha M$, there exists $c \in M$ such that $c\alpha(e\alpha b) = e$. Thus $(e\alpha c)\alpha(e\alpha b)= e\alpha e$. Therefore $(e\alpha c)\alpha(e\alpha b) = e$. Hence e is the left zeroid of $e\alpha M$ and $e\alpha c$ is the left inverse of $e\alpha b$. Thus $e\alpha M$ is a group. Hence (g, Γ) is a soft group over M. □

Let M be a $\Gamma-$semigroup and $\alpha \in \Gamma$. and $a \in M$ Define a mapping $* : M\times M \rightarrow M$ such that $b * a = b\alpha a$, for all $b \in M$. Then $(M, +, *)$ is a subsemigroup of M. It is denoted by $M\alpha a$. Define $h : \Gamma \rightarrow \mathbb{P}(M)$, $\mathbb{P}(M)$ is a power set of M by $h(\alpha) = M\alpha a$. Then (h, Γ) is a soft semigroup over M.

Theorem 3.8. *Let M be a $\Gamma-$semigroup. If e is a strongly idempotent left zeroid then (h, Γ) is a soft regular semigroup over M.*

Proof. Obviously e is the right identity of the soft semigroup (g, Γ). Suppose $\alpha \in \Gamma$ and $z\alpha e \in M\alpha e$. Then there exists $g \in M$ such that $g\alpha z\alpha e = e$. Consider $e = e\alpha e = e\alpha(g\alpha z\alpha e) = (e\alpha g)\alpha(z\alpha e)$. Therefore e is a left zeroid of $M\alpha e$. Suppose $x \in M\alpha e$.Then there exists $y \in M\alpha e$ such that $y\alpha x = e$. Then by Theorem[3.15] $x\alpha y\alpha x = x\alpha e = x$. Thus $M\alpha e$ is a regular semigroup.Hence (h, Γ) is a soft regular semigroup over M. □

4. Conclusion

Molodtsov [**12**] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Aktas and Cagman [**2**] defined the notion of soft groups. In 2008, F. Feng et al. [**5**] defined soft semirings and several related notions to establish a connection between soft sets and semirings. Soft rings are defined by Acar et.al [**1**]. Jayanth Ghosh et.al [**9**] initiated the study of fuzzy soft rings and fuzzy soft ideals. In this paper, we studied $\Gamma-$semigroup M. as a class of soft semigroups over M.

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