Bulletin of International Mathematical Virtual Institute
ISSN 1840-4359
Vol. 1(2011), 39-43

Former
Bulletin of Society of Mathematicians Banja Luka
ISSN 0354-5792 (p), ISSN 1986-521X (o)

COMPLEMENT FREE DOMINATION NUMBER OF A BIPARTITE GRAPH

Y.B.Venkatakrishnan[1], V.Swaminathan[2]

Abstract

Let $G = (X, Y, E)$ be a bipartite graph. A subset S of X is a complement free dominating set if S is an $X-$dominating set and $X - S$ is not an $Y-$dominating set. A subset S of X is called a minimal complement free dominating set if any proper subset of S is not a complement free dominating set. The minimum cardinality of a minimal complement free dominating set is called the complement free domination number of G and is denoted by $\gamma_{cf}(G)$. In this paper some results on complement free domination number are obtained.

AMS Mathematics Subject Classification (2010): 05C69.

Key words and phrases: bipartite graph, complement free dominating set, $Y-$dominating set, $X-$dominating set.

1 Introduction

Let $G = (V, E)$ be a connected undirected graph. The neighbourhood of a vertex $v \in V$ in G is the set $N_G(v)$ of all vertices adjacent to v in G. For a set $D \subseteq V$, the open neighbourhood $N_G(D)$ is defined to be $\cup_{u \in D} N_G(u)$ and the closed neighbourhood $N_G[D] = N_G(D) \cup D$.

A set $D \subseteq V$ is a dominating set of G if $N_G[D] = V$. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G. A dominating set D is called a split dominating set [2] if $\langle V - D \rangle$ is disconnected. The split domination number of G, denoted by $\gamma_s(G)$, is the minimum cardinality of a split dominating set of G. A dominating set D is called a nonsplit dominating set [3] if $\langle V - D \rangle$ is connected. The nonsplit domination number of G, denoted by $\gamma_{ns}(G)$, is the minimum cardinality of a nonsplit dominating set of G. In a similar fashion, the concept of complementary tree domination number of a graph G [5] and complementary nil domination

[1]Department of Mathematics, School of Humanities and Sciences, SASTRA University,Tanjore, India e-mail: venkatakrish2@maths.sastra.edu

[2]Reader(Retd) Ramanujan Research Center, Saraswathi Narayanan College,Madurai, India. e-mail: sulanesri@yahoo.com

number of a graph G [8] was defined. The varities of dominating parameters were described in [1].

Stephen Hedetniemi ([6],[7]) and Renu Laskar proposed bipartite theory of graphs, in which concepts in graph theory have equivalent formulations as concepts for bipartite graphs. Equivalently, given any problem say P, on an arbitrary graph G, there is very likely a corresponding problem Q on a bipartite graph G_1, such that a solution for Q provides a solution for P. One such reformulation is the concept of $X-$dominating set and $Y-$dominating set.

Unless otherwise stated, we will consider bipartite graph $G = (X, Y, E)$ with no multiple edges. Two vertices u, v in X are X-adjacent if they are adjacent to a common vertex in Y. A subset D of X is an X-dominating set [6] if every vertex in $X - D$ is X-adjacent to at least one vertex in D. The $X-$domination number of G,denoted by $\gamma_X(G)$, is the minimum cardinality of a X-dominating set of G. A subset $S \subseteq X$ which dominates all vertices in Y is called an Y-dominating set [6] of G. The Y-domination number denoted by $\gamma_Y(G)$ is the minimum cardinality of a Y-dominating set of G. A subset S of X is hyper independent [6] if there does not exist a vertex $y \in Y$ such that $N_G(y) \subseteq S$. The hyper independence number of G, denoted by $\beta_h(G)$, is the maximum cardinality of a hyper independent set of G.

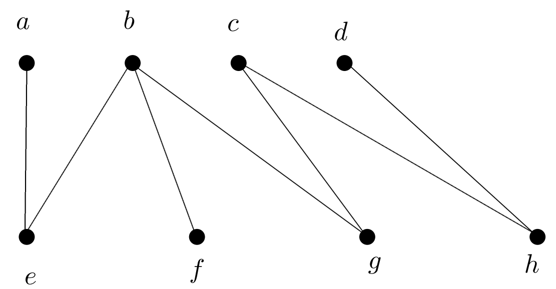
Let $S \subseteq X$ and let $u \in S$. A vertex $v \in X - S$ is called a Y-private neighbor of u with respect to S if u is the only point in S such that u and v have common adjacent point in Y. The X-corona of a bipartite graph H is the bipartite graph $G = (X^1, Y^1, E^1)$ where $X^1 = X \cup \{u_1, u_2, \cdots, u_p\}$ and $Y^1 = Y \cup \{v_1, v_2, \cdots, v_p\}$ and every vertex in X is X-adjacent to a unique u_i through v_i,$1 \leq i \leq p$.The complement of G [5] denoted by $\overline{G} = (X, Y, E^")$ is defined as follows: (i) No two vertices in X are adjacent. (ii) No two vertices in Y are adjacent. (iii) $x \in X$ and $y \in Y$ are adjacent in $\overline{G}$ if and only if $x \in X$ and $y \in Y$ are not adjacent in G.

Similar to the idea of complementary tree dominating set and complementary nil dominating set defined in arbitrary graph, we define complement free domination in bipartite graph.

2 Complement free dominating set

Definition 2.1 *A subset S of X is called a complement free dominating set if S is an $X-$dominating set and $X - S$ is not an $Y-$dominating set. The complement free domination number of G, denoted by $\gamma_{cf}(G)$, is the minimum cardinality of a complement free dominating set of G.*

Example 2.1



$S = \{a, d\}$ is a $X-$dominating set but not a complement free dominating set. The set $D = \{b, c\}$ is a complement free dominating set.

Remark 2.1 *If Y contains an isolated vertex, then any $X-$dominating set will be a complement free dominating set. Therefore, hereafter, by a graph G we mean a bipartite graph $G = (X, Y, E); |X| = p$, without loops, multiple edges and with no isolated vertex in Y.*

Remark 2.2 $\gamma_X(G) \leq \gamma_{cf}(G)$.

Proposition 2.1 *Let G be a graph, every $\gamma_{cf}-$set intersects with every $Y-$dominating set.*

Proof: Let D be a $\gamma_{cf}-$set and D_1 be a γ_Y-set of G. Suppose $D \cap D_1 = \phi$, then $D_1 \subseteq X - D$. $X - D$ contains a $Y-$dominating set D_1. Therefore, $X - D$ itself is a $Y-$dominating set, which is a contradiction. ■

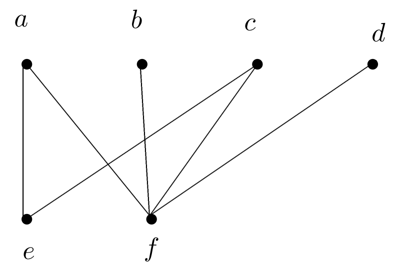
Theorem 2.1 *Let G be a graph. A $X-$dominating set S is a complement free dominating set if and only if S is not a hyper independent set.*

Proof: Let S be a complement free dominating set. Then, $X - S$ is not an $Y-$dominating set. Therefore, there exists $y \in Y$ such that it is not adjacent to any vertex in $X - S$. Equivalently, there exists $y \in Y$ such that $N(y) \subseteq S$. Therefore, S is not a hyper independent set of graph G.

Conversely, let S be a $X-$dominating set which is not a hyper independent set. That is, there exists $y \in Y$, such that $N(y) \subseteq S$. Therefore, $y \in Y$ is not adjacent to a vertex of $X - S$. Hence, $X - S$ is not an $Y-$dominating set. Therefore, S is a complement free dominating set. ■

Remark 2.3

Complement of a minimal complement free dominating set need not be a complement free dominating set. consider the graph



$S = \{a, c\}$ is a minimal complement free dominating set but $X - S = \{b, d\}$ is not a complement free dominating set.

Theorem 2.2 *Let S be a complement free dominating set of a graph G. Then S is minimal if and only if for each vertex $u \in S$ one of the following conditions is satisfied:*
(i) there exists a $v \in X - S$ such that v is a Y-private neighbor of u with respect to S.
(ii) $X - (S - \{u\})$ is a $Y-$ dominating set of G.

Proof: Suppose S is a minimal complement free dominating set. Then $S-\{u\}$ is not a complement free dominating set. That is, $S - \{u\}$ is not a $X-$dominating set or $X - (S - \{u\})$ is a $Y-$ dominating set of G. If $S - \{u\}$ is not a $X-$dominating set, there exists $v \in X-(S-\{u\})$ not $X-$adjacent to a vertex in $S-\{u\}$ but $X-$adjacent to a vertex in S. Therefore, we get (i). If $X-(S-\{u\})$ is a $Y-$dominating set of G, which is (ii).

Conversely, assume conditions (i) and (ii) hold. Let S be a complement free dominating set. Let us assume S is not minimal complement free dominating set. Then, there exists a vertex $u \in S$ such that $S - \{u\}$ is a complement free dominating set. Equivalently, $S-\{u\}$ is a $X-$dominating set and $X-(S-\{u\})$ is not a $Y-$dominating set. If $X - (S - \{u\})$ is not a $Y-$dominating set of G, we get a contradiction to (ii). Also every vertex in $X - S$ is X-adjacent to at least one vertex in $S - \{u\}$, so condition (i) does not hold for u. ■

Observation 2.1 *(i)$\gamma_{cf}(K_{m,n} - \{e\}) = m - 1$, where e is an edge in $K_{m,n}$.*
(ii) $\gamma_{cf}(\overline{mK_2}) = m - 1$ for $m \geq 3$.

3 Bounds for complement free domination number

Let $\delta_X(G) = min\{d(y) : y \in Y\}$.

Observation 3.1 $\gamma_{cf}(G) \geq \delta_X(G)$.

Proof: Let S be a complement free dominating set. Then, $X - S$ is not a $Y-$dominating set. There exists a vertex $y \in Y$ not dominated by a vertex of $X - S$. Since, every vertex in Y should have degree at least $\delta_X(G)$. Therefore, $\gamma_{cf}(G) \geq \delta_X(G)$. ■

Theorem 3.1 *For any graph G, $\gamma_{cf}(G) \leq \gamma_X(G) + \delta_X(G) - 1$.*

Proof: Let S be a γ_X-set of G. Let $y \in Y$ be a vertex such that $d(u) = \delta_X(G)$. Then atleast one vertex $x \in N(y)$ must be in γ_X-set S, say $x \in S$. Now $S \cup (N(y) - \{x\})$ is a complement free dominating set. Therefore, $\gamma_{cf}(G) \leq |S \cup (N(y) - \{x\})| \leq \gamma_X(G) + \delta_X(G) - 1$. ■

Remark 3.1 *The upper bound in theorem 3.1 is attained in the graphs (i)$K_{m,n}$ (ii) X-Corona of $K_{m,n}$.*

Theorem 3.2 *For any connected graph G, if $\gamma_Y(G) = k$, then $\gamma_{cf}(G) \leq p - k + 1$.*

Proof: Let S be a γ_Y-set of G. Then $S - \{u\}$ is not a $Y-$dominating set of G. Since, S is a $Y-$dominating set, $X - S$ is a X-dominating set of G. Therefore, $(X - S) \cup \{u\}$ is a $X-$dominating set such that $S - \{u\}$ is not a $Y-$dominating set. Therefore, $\gamma_{cf}(G) \leq p - k + 1$. ■

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Received by the editors July 1, 2011.